

DESIGN AND IMPLEMENTATION OF MINIMUM TIME COMPUTER CONTROL SCHEMES
FOR START-UP OF A DOUBLE EFFECT EVAPORATOR

By

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NOMENCLATURE

A	= Heat transfer area, ft^2
C	= Solute concentration, $\text{lbs}/(\text{lb solution})$
\hat{C}	= Specific heat, Btu/lb
D	= Diameter, ft
G	= Mass velocity, $\text{lbs}/(\text{ft}^2)(\text{hr})$
Gr	= Grashof number, dimensionless
H	= Hold-up, lbs
I	= Index set
K	= Index set
L	= Length, ft
N	= Number of tubes
P	= Pressure, $\text{lbs}/(\text{in}^2)$
$P(t)$	= Covariance of estimate, vector
Pr	= Prandtl number, dimensionless
Q	= Heat transfer rate, $\text{Btu}/(\text{hr})$
$Q(t)$	= Covariance of process noise, vector
$R(t)$	= Covariance of measurement noise, vector
Re	= Reynold's number, dimensionless
T	= Temperature, $^{\circ}\text{F}$
ΔT	= Temperature difference, $^{\circ}\text{F}$
U	= Overall heat transfer coefficient, $\text{Btu}/(\text{hr})(\text{ft}^2)(^{\circ}\text{F})$
$U(t)$	= Control vector
$V(t)$	= Process noise, vector
V'	= Vapor volume, ft^3
V	= Vapor flow rate, lbs/min

W	= Liquid flow rate, lbs/min
X	= State variables, vector
\hat{X}	= Estimate of state variables, vector
X_{tt}	= Lockhart-Martinelli factor
Y	= Calculated observations, vector
\hat{Y}	= Actual observations, vector
f	= Function of
g	= Acceleration due to gravity, $\text{ft}/(\text{hr}^2)$
h	= Liquid enthalpy, Btu/lb
h^v	= Vapor enthalpy, Btu/lb
\hat{h}	= Film coefficient, $(\text{Btu})(\text{ft})/(\text{hr})(\text{ft}^2)(^\circ\text{F})$
k	= Thermal conductivity, $(\text{Btu})(\text{ft})/(\text{hr})(\text{ft}^2)(^\circ\text{F})$
p	= Variance of estimate
t	= Time, minutes
u	= Control variable
v	= Process noise
w	= Measurement noise
x	= State variable
\hat{x}	= Estimate of state variable

Subscripts

o	= Outside world
1	= First effect
2	= Second effect
$i\ j$	= From unit j to unit i
a	= Before
c	= Condensate

s	=	Sensible heating zone
t	=	Tube
B	=	Boiling zone
w	=	Wall conditions
F	=	Feed
i/j	=	At "i" given conditions at "j"
f	=	Film conditions or final condition

Superscripts

i	=	Inside
o	=	Outside
v	=	Vapor
*	=	Optimality

Greek Letters

α	=	Point constraint multiplier
β	=	Inequality constraint multiplier
Θ	=	Estimated parameters, vector
θ	=	Estimated parameter
ρ	=	Density, lbs/(ft ³)
λ	=	Lagrange multiplier or latent heat of vaporization, Btu/lb
μ	=	Viscosity, lbs/(ft)(hr)

Abstract of Dissertation Presented to the
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Major Department: Chemical Engineering

The application of optimal control theory to a chemical engineering problem is investigated by the development and implementation of a control policy for the minimum time start-up of a double effect evaporator.

The particular evaporator on which experimental runs were made was a laboratory scale double effect evaporator with reverse feed. It was completely instrumented for control by the installation of orifices for measuring flow rates, thermocouples for measuring temperatures, pressure taps for measuring pressures and hold-ups, and pneumatic control valves for manipulating flow rates. Transducing and controlling instruments were installed. In order to do on-line computerized data logging and control, interfacing of the process with the IBM 370/165 computer on campus was provided by an existing IBM 1070 interface. A part of the existing user circuitry associated with the interface had to be rewired and modified to function appropriately.

The dynamic model of the evaporator consisted of six differential or state equations and about sixty algebraic equations. This latter

group consisted of connection equations between the effects, property correlations and heat transfer equations. To overcome the uncertainty in the empirical relationships for the inside film coefficient two unknown parameters were introduced, one for each effect. These parameters were estimated by correlating model predictions with data collected on experimental runs. A nonlinear least-squares technique was utilized to get the best fit.

The algorithm used for the theoretical development of a minimum time policy is one in which Hamiltonian minimizations result in control policy updates on successive iterations. Control variable constraints and point constraints are accounted for along the trajectory. The utility of the algorithm is enhanced by assuming a control scenario and determining whether it is optimal when compared to other likely scenarios. This approach keeps the number of active state and adjoint variables to a minimum at any particular time resulting in simple Hamiltonians and less computational expense for the integration of the state and adjoint equations and Hamiltonian minimizations.

The minimum time algorithm was used on the evaporator model to determine the optimal policy under three different sets of conditions. In the first case it was assumed that there was a constraint on the maximum value of the second effect hold-up. The second case dealt with a different set of control variables in that the feed rate to the second effect was assumed to be fixed. In the third case the assumption of a constrained maximum second effect hold-up was done away with. The simulation results indicated that the third case resulted in the smallest start-up time with the optimal policy calling for an overfilling of the second effect followed by a gradual decrease in the second effect hold-up

to the desired value which took place when boiling just started in the second effect.

For all three cases it was found that the control policy is bang-bang in nature and that the control switches occur at times at which the point constraints are met on the assumed scenario. Because of this the switching times can be related to the state variables and a feedback control policy is obtained.

Experiments were run to try out the optimal control policy and to test the model. On an average, the simulations resulted in final times which were between ten and fifteen percent within those obtained experimentally. This accuracy was reasonable considering the experimental problems associated with hold-up measurements and analog control of the hold-ups and the theoretical problems associated with the assumption of the heat transfer mechanisms.

CHAPTER I

INTRODUCTION

Optimal control theory has been developed to a fairly sophisticated level in the field of electrical engineering. However, the uses of the theory and possible applications in chemical engineering have been virtually unexplored. The reasons for this rather limited progress on both the theoretical and applied fronts are many (Foss, 1973).

The starting point in the applications of control theory is a good dynamic model of the process. Most chemical processes have been modeled poorly due to an incomplete understanding of the complex interactions among numerous variables. High dimensionality and nonlinearities in behavior require the use of sophisticated numerical techniques for the simulation and design of control schemes. Many chemical processes have inherently large time constants which make them unsuitable for control. It is generally not possible to make all the measurements that are required for a feedback control scheme, and, in addition, measurements are subject to noise which is not readily filtered in nonlinear systems. Since most of the states are not measurable, there is a need for building reduced order observers which again is a formidable problem for nonlinear systems.

In spite of the above drawbacks, quite a few articles on the subject of optimal control have appeared in the chemical engineering

literature during the last five years. The early investigations dealt with the control of simplified lumped parameter linear processes. The linear quadratic loss problem resulted in feedback control which was particularly useful in regulatory control; i.e. control in the face of disturbances (Nieman and Fisher, 1970; Newell and Fisher, 1971; Newell et al., 1972). The arbitrary nature of the correlation of the weighting matrices to the actual physical problem often makes the linear quadratic loss problem unrealistic. Later investigators extended these techniques to the control of nonlinear lumped parameter systems by using various forms of linearization on the system equations (Weber and Lapidus, 1971a, 1971b; Siebenthal and Aris, 1964; Tsang and Luus, 1973). Others worked on nonlinear systems with one or two control and state variables (Joffe and Sargent, 1971; Jackson, 1966). The control of distributed parameter systems is still in its infancy. Some investigators have reported sub-optimal control of distributed systems in which some other criterion, such as minimization of a Lyapunov functional, is used (Vermeychuk and Lapidus, 1973a, 1973b; Chant and Luus, 1968). Simulated start-up studies have been made on plate distillation columns (Pollard and Sargent, 1966) and autothermic reaction systems (Jackson, 1966).

This work was directed towards applying existing control theory to a useful chemical process. The aim was to study the dynamics and control of a simple, yet reasonably complex, piece of equipment commonly found in the chemical industry. A double effect evaporator was chosen as the subject of study for the above reason and also because a laboratory scale double effect evaporator was available for experimental work. The study involved,

a) Developing a nonlinear model for the evaporator.

- b) Estimating model parameters to fit experimental data.
- c) Developing a minimum start-up time control policy taking into account constraints on the state and control variables and putting the optimal policy in feedback form in terms of switching times.
- d) Experimentally determining the effect of the policy.

This approach differs from previous ones in a few respects. The model is highly nonlinear and is treated as such. No linearization is resorted to as start-up involves large changes in the state variables and linearized equations would be inaccurate. The mathematics involved in obtaining the minimum time policy is simplified as the approach adopted presupposes a start-up scenario and then verifies that it is optimal. The algorithm leading to the optimal policy handles constraints on control and state variables in a logical fashion by directly holding the state or control variable on the constraint and changing the equation set and its solution procedure as a result. This avoids the use of penalty function methods and the like. Finally, the control policy is experimentally verified.

The minimum time objective was chosen primarily from the point of view of economics. Control costs during start-up are minimal compared to the start-up time in batch processes. Reducing the start-up time results in reduced down time thus improving cycle efficiency and increasing profits. The food industry is an example of an industry which must shut down frequently to have the processing equipment cleaned. Orange juice is concentrated in multiple effect evaporator systems, and these systems are cleaned about three times a day. A second reason for minimum time start-up was more specific to the ultimate use of the particular double effect evaporator investigated. It is to be used in an

undergraduate laboratory experiment in computer control, and past experiences indicated that it took a very long time to bring it to steady state under manual control. Thus, in order to reduce the start-up time and consequently to reduce the amount of on-line computer time for steady state observations, it was imperative to have start-up in a minimum time.

Chapter II contains a description of the experimental evaporator, the instrumentation and the interfacing equipment with the IBM 370/165 (which is the main computer on campus). Chapter III deals with the building of a dynamic model for the evaporator and also the estimation of parameters from experimental data. A derivation of the optimal control algorithm is given in Chapter IV. It also contains the simulated and experimental results of the application of the control algorithm. Some comments and proposals for further work are given in Chapter V. Appendix A contains all the heat transfer equations which supplement the main model equations in Chapter III. A listing and description of the computer program implementing the optimal control algorithm is the subject of Appendix B.

CHAPTER II

DESCRIPTION OF EVAPORATOR AND COMPUTER INTERFACING EQUIPMENT

II.1 Evaporator Layout and Description

The double effect evaporator is located in the unit operations laboratory of the chemical engineering department. Figure 2.1 is a schematic of the double effect, showing the arrangement of the two effects, EV1 and EV2, and the basic process and vapor lines. Note that backward feed is used; that is, the vapor flow and process fluid flow are in opposite directions.

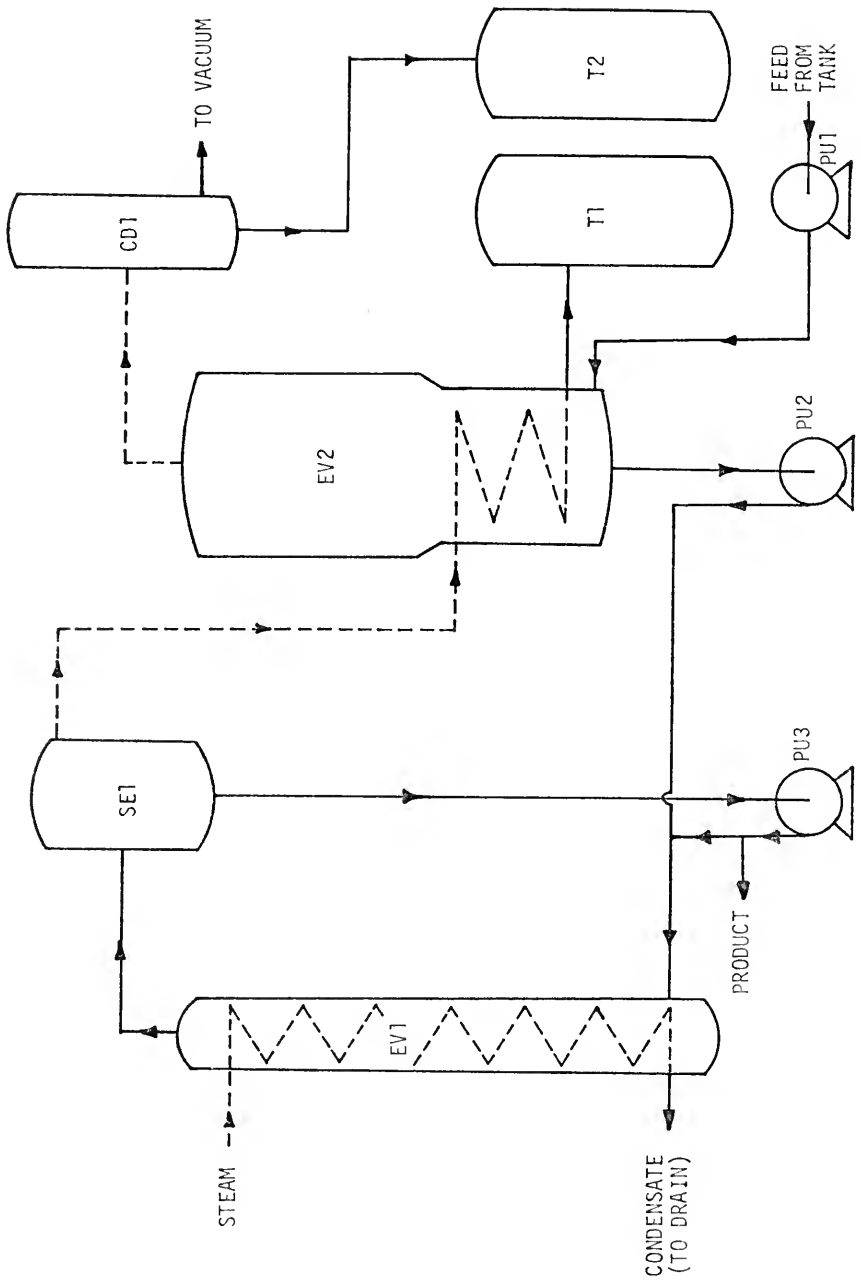
The first effect is a long tube vertical (LTV) evaporator. It contains 3 tubes, each 9 feet, 6 inches (2.90 m) long and 1 inch (0.0254 m) O.D. with heating steam at about 20 psig (2.39 bar) on the outside of the tubes. The process fluid flows upward through the tubes either by natural or forced circulation. The latter method is almost always used because of the increased heat transfer coefficients obtainable. The pressure on the process side is at or slightly above atmospheric.

The mixture of process fluid and vapor formed in the first effect enters a vapor-liquid separator, SE1, which is at the same pressure as the first effect. The liquid is drawn off the bottom of the separator and is recirculated back into the first effect by pump PU3 after some liquid product is withdrawn. Fresh feed to the first

Legend

EVn	Evaporator effect
SEn	Vapor-liquid separator
CDn	Vapor condenser
Tn	Condensate tank
PUn	Pump

Figure 2.1 Schematic Diagram of the Double Effect Evaporator



effect is pumped by pump PU2 from the second effect. The vapor from separator SE1 is used as the steam input in the second effect. This leads to steam economy as one pound of heating steam used in the first effect should evaporate more than one pound of water from the first and second effects combined.

The second effect, EV2, is a calandria type effect in which there are 15 tubes, each 2 feet, 4 inches (0.711 m) long and 1 inch (0.0254 m) O.D. The effect also has a 2 inch (0.0508 m) O.D. central downtake. Heat transfer is by natural convection only, resulting in much lower heat transfer coefficients compared to the first effect. Fresh preheated feed is pumped into the bottom of the second effect from the feed tank by pump PU1. The heating medium is the vapor from the first effect on the outside of the tubes. Above the calandria is a vapor body which separates the vapor from the liquid. The vapor is drawn into a condenser, CD1, by means of a vacuum produced by a steam jet ejector. The ejector maintains the pressure on the process side in the effect at around 10 inches mercury vacuum (0.675 bar).

The vapor condensate from the first effect is collected in tank T1 and that from the second effect is collected in tank T2, both of which are maintained at a vacuum by the same steam jet ejector.

II.2 Operating Notes

There were a few precautions which had to be observed during operation.

- 1) The feed rate was kept at around 2-3 gpm (0.12 to 0.18 kg/s).
- 2) The recirculation rate in the first effect was kept at a maximum of 15 gpm (0.9 kg/s). A higher rate caused entrainment of liquid with

the vapor in the vapor-liquid separator SE1. This separator has no baffling of any kind and is very inefficient at high flow rates.

3) To avoid cavitation in the recirculation pump PU3, care was taken to see that the vertical suction leg from the separator to the recirculation pump was always filled with liquid. This was particularly critical when the pump was first started. Incomplete filling of the vertical leg led to pulsating flows resulting in large upsets in the evaporator operation. A recirculation rate higher than 15 gpm (0.9 kg/s) also caused a high discharge head on pump PU3, much higher than the maximum discharge head on pump PU2 (which is of a smaller capacity), eliminating all flow of fresh feed to the first effect.

4) The liquid level in the second effect was maintained around the top of the tubes for best utilization of the heat transfer area.

II.3 Evaporator Instrumentation

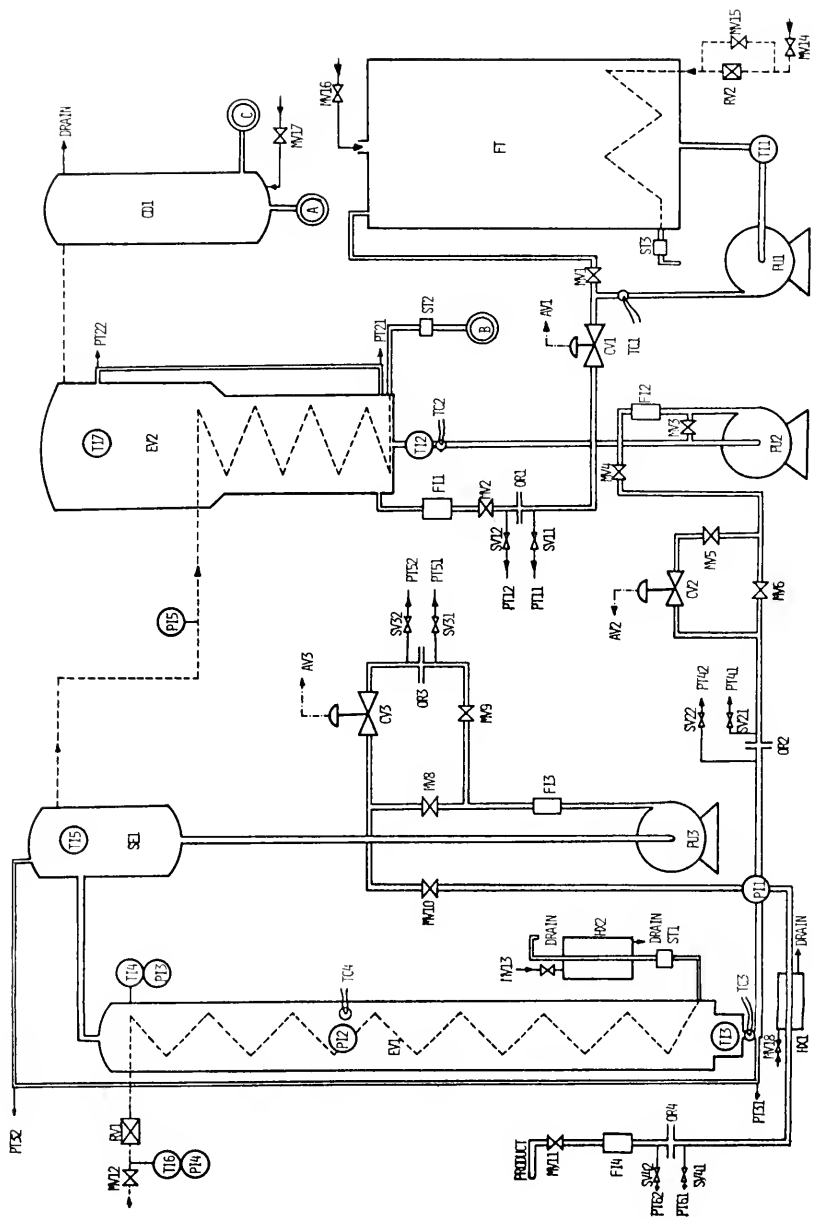
As part of this work the evaporator piping had to be modified to accomodate the instrumentation required for control. The work consisted mainly of installing pneumatic control valves, orifices, pressure taps, thermocouples and extra manual valves. Figures 2.2 and 2.3 show the detailed instrumentation of the evaporator. The legend of Figure 2.2 also applies to Figure 2.3.

Three pneumatic control valves, CV1, CV2 and CV3, were installed in the feed, inter-effect and recirculation lines respectively. CV2 and CV3 were normally closed (air-to-open) valves and were installed in bypasses on the lines, whereas CV1 was a normally open valve and was installed in the feed line as such. The purpose of the by-passes was to

Legend

<u>Valves</u>		<u>Other</u>
MVn		Pressure tap
RVn		Pump
SVnm		Heat exchanger
		Evaporator effect
CVn		Vapor-liquid separator
		Vapor condenser
		Feed tank
		Steam ejector
		Storage tank
		Steam trap
		Process liquid line
		Steam (vapor) line
		Air line
<u>Indicators</u>		
PI n		
TI n		
FI n		
OR n		
TC n		

Figure 2.2 Evaporator Instrumentation



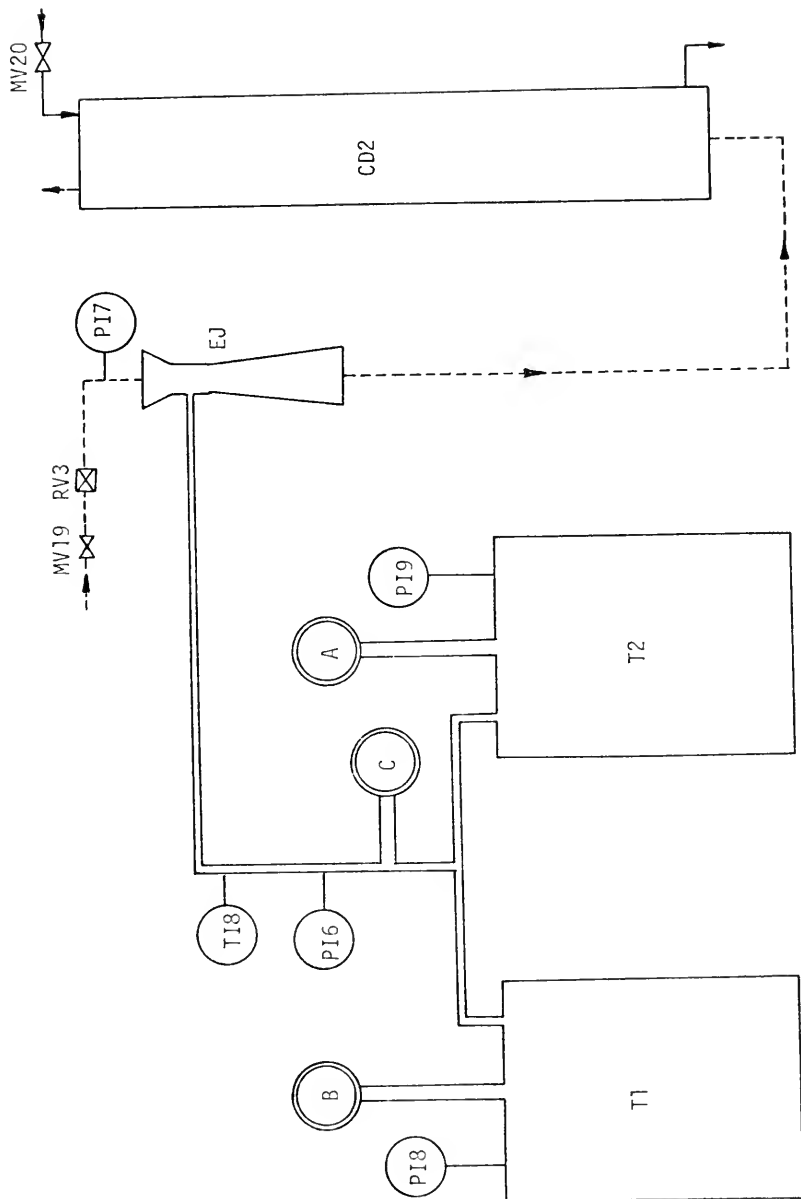


Figure 2.3 Evaporator Instrumentation

allow for complete or partial manual control of experimental runs when desired.

Flow rates were measured with square-edged orifices, OR1, OR2, OR3 and OR4, installed in the feed, inter-effect, recirculation and product lines respectively. The pressure drop across an orifice is indicative of the flow through it.

Liquid levels (proportional to hold-ups) were measured by taking the difference in total pressure between the bottom and top of each of the two effects. Pressure taps were installed at both ends of the sight glasses for this purpose. The upper taps were also used to measure absolute pressure in the effects.

Temperatures were measured by jacketed copper-constantan thermocouples, TC1, TC2, TC3 and TC4. These were installed in the feed line, at the exit of the second effect, at the entrance to the first effect, and in the steam chest of the first effect, severally.

All liquid lines from the pressure taps and air lines to the valves were brought to a central panel in the front of the evaporator with 1/4-inch poly-flo tubing. Quick-connect fittings were used at the panel so that leads to the interfacing equipment could be connected quickly when required. The thermocouple wires also terminated with special thermocouple outlets at the panel.

II.4 Transducing and Controlling Equipment

The transducing and controlling equipment was installed in a 19-inch relay rack on casters. All air and liquid lines were of poly-flo tubing with quick-connect fittings. This made the rack very versatile as it can be moved to a number of different pieces of equipment

if desired. A layout of the cabinet is shown in Figure 2.4.

The pneumatic controllers have adjustable proportional and reset action, motorized set point control, and indication facilities. One Fischer and Porter model 51 and three Taylor model 662R controllers were installed. The set point motor of the Fischer and Porter controller operates on a pulse train input and Taylor controllers on a 24-volt DC signal. The controllers are equipped with feedback potentiometers which indicate their set point positions. The pneumatic input signal range to each controller (from the DP cells) is 3-15 psig and that of the output pressure to the associated valve is also a 3-15 psig signal.

The EMF to pneumatic converters (not used in the current experiments) are Foxboro model 33A converters. They can transduce either a millivoltage or voltage signal into a 3-15 psig pneumatic signal.

The differential pressure (DP) cells used are Foxboro Model 13A DP cells. The adjustable range of the differential input signal is 0-500 inches water and the proportional pneumatic output is in the 3-15 psig range. These DP cells were used to transduce the pressure drops across the orifices and the pressure differences corresponding to the liquid levels. The outputs were thus proportional to the flow rates or to the liquid levels (hold-ups). The DP cells were also used for measuring absolute pressures by venting the high pressure side when measuring pressures above atmospheric. The output pressure in this case was proportional to the vacuum or above-atmospheric pressure.

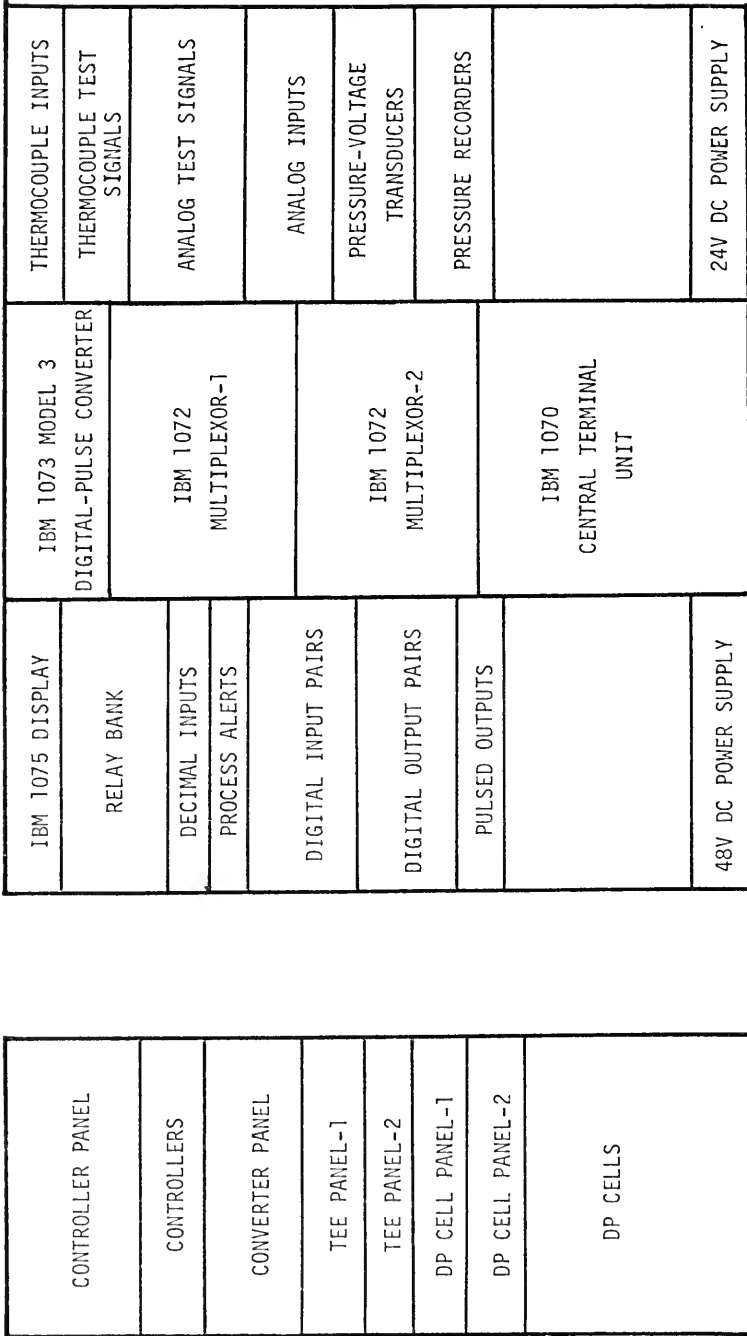


Figure 2.4 Layout of Transducer and Controller Cabinet and IBM 1070 Cabinets

II.5 IBM 1070 Interface

An IBM 1070 interface was used which consisted of an IBM 1071 central terminal unit, two IBM 1072 multiplexing units, an IBM 1073 Model 3 digital to pulse converter and an IBM 1075 decimal display. The interface with all the auxiliary equipment resides in 3 relay racks as shown in Figure 2.4. These cabinets were originally assembled by R. C. Eschenbacher and are described in his Ph.D. thesis (Eschenbacher, 1970). However, as part of this work, some of the equipment had to be rewired to accomodate pulse duration outputs and most of the relays had to be rewired and changed to double coil relays in order to isolate the IBM equipment from the user circuitry. The entire set up is described in detail in the GIPSI (General Interface for Process Systems Instrumentation) hardware manual (GIPSI, 1973) and is summarized very briefly here.

1) Input

The pressure-voltage transducers converted the 3-15 psig air signals from the DP cells to 0-5 volt DC signals which were fed as analog inputs to the 1070. The millivoltage thermocouple signals were also fed as analog inputs through the special thermocouple input feature in the 1070. The unit also has a facility for digital input which was not used.

Another convenient input facility frequently used was a form of digital input through pre-designated demand functions which were dialled into the system through rotary switches.

2) Output

Output from the 1070 was in digital and in pulse form. The digital output was used mainly to ring a bell to alert the operator

to possible alarm conditions in the hardware and software. The pulsed output was obtained from the 1073 and was used to move the set points on the controllers. The digital to pulse converter (1073) has outputs in the form of a pulse train as well as a duration pulse.

The 1075 decimal display is a feature which utilizes digital outputs and was used to display particular variable values or error codes.

3) Process Alerts

The 1070 interface is linked with the IBM 370/165 computer on campus. The seven process alerts attached to a process alert (PA) bus in the 1070 provide a hardware interrupt capability of the computer by the process. The software issues a conditional read of the 1070 terminal to the computer. It is then in a hardware wait stage. When the PA bus is activated, by one of the process alerts on the 1070, the IBM 370/165 computer senses the closure and reactivates the software which then determines which PA was set. The software then resets the PA and executes the program associated with the PA. Process alert 1 has the added facility of being set automatically by a hardware poller on which the timing is adjustable. This enables one to have PA1 periodically and automatically set after a predetermined time interval has elapsed.

Figure 2.5 is a schematic illustrating the flow of information among the various hardware components of the experiment.

II.6 Software

The GIPSI software package was originally written by R. C. Eschenbacher. Version 2, the version used, was written by L. A. Delgado

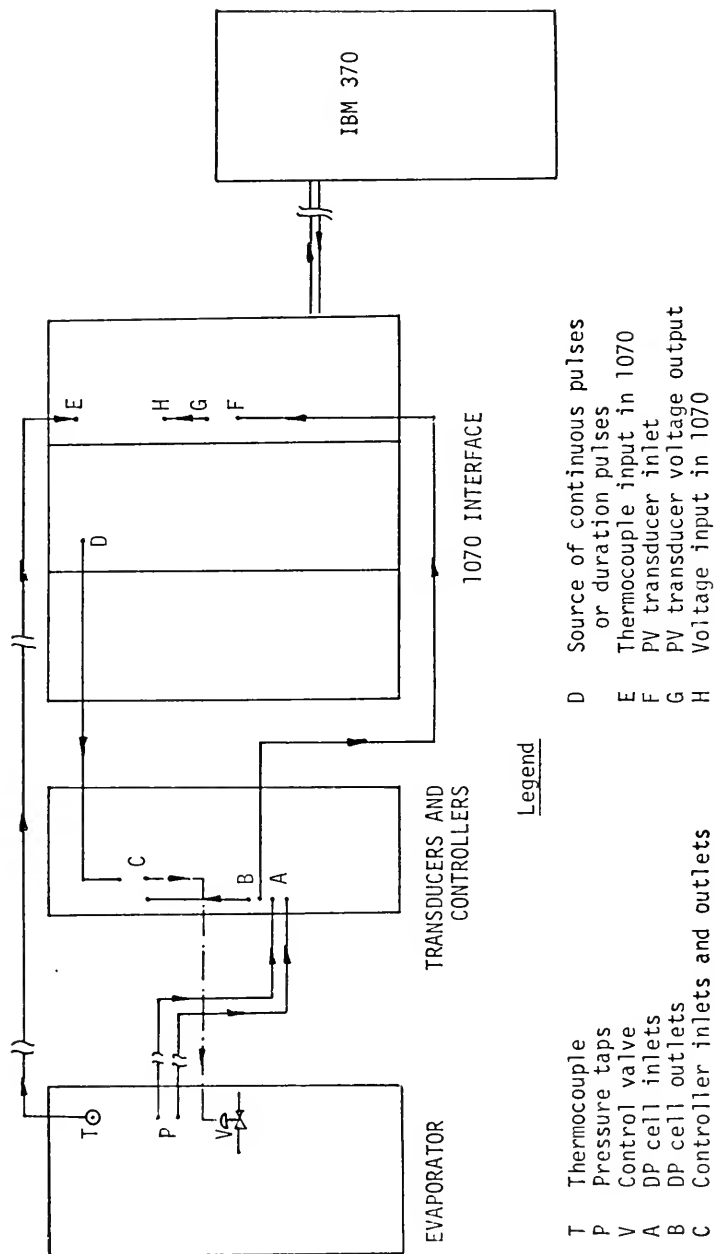


Figure 2.5 Process - Interface - Computer Information Flow

(GIPSI, 1973) and was marginally modified as part of this work to extend its capabilities. It is written entirely in Fortran (with the exception of certain input/output routines in BTAM provided by IBM), has an extensive debugging facility, and an extensive error handling facility to flag user software and hardware errors. A simplified flow chart is reproduced from Westerberg and Eschenbacher (1971) and is shown in Figure 2.6. It is described in greater detail in Westerberg and Eschenbacher (1971) and GIPSI (1973).

The heart of the software is the concept of the execute and delay stacks. When a PA is set, it is identified and the program (or programs) associated with it are stacked by the program stacker in the order of priority on the execute stack. Control then passes to the Execute subprogram which then examines the execute stack and passes control one at a time to the programs that are due for execution. If the sequence began with PA1, its response program CLOCK removes programs from the delay stack if their delay time has expired and puts them on the execute stack. The Execute subprogram then finds additional programs on the execute stack which it continues to remove and cause to be executed. This is done until the execute stack is empty whence control returns to the PA handler which issues a conditional read and the IBM 370/165 again waits for a PA to be set to start the cycle again. All delay times are compared to the computer clock. Data on program priorities, delay time, etc. are specified in the Program Descriptive Data.

All the user has to do is to provide his specific programs associated with the various process alerts, the program descriptive data for all his programs, and a subroutine GOTO which the Execute

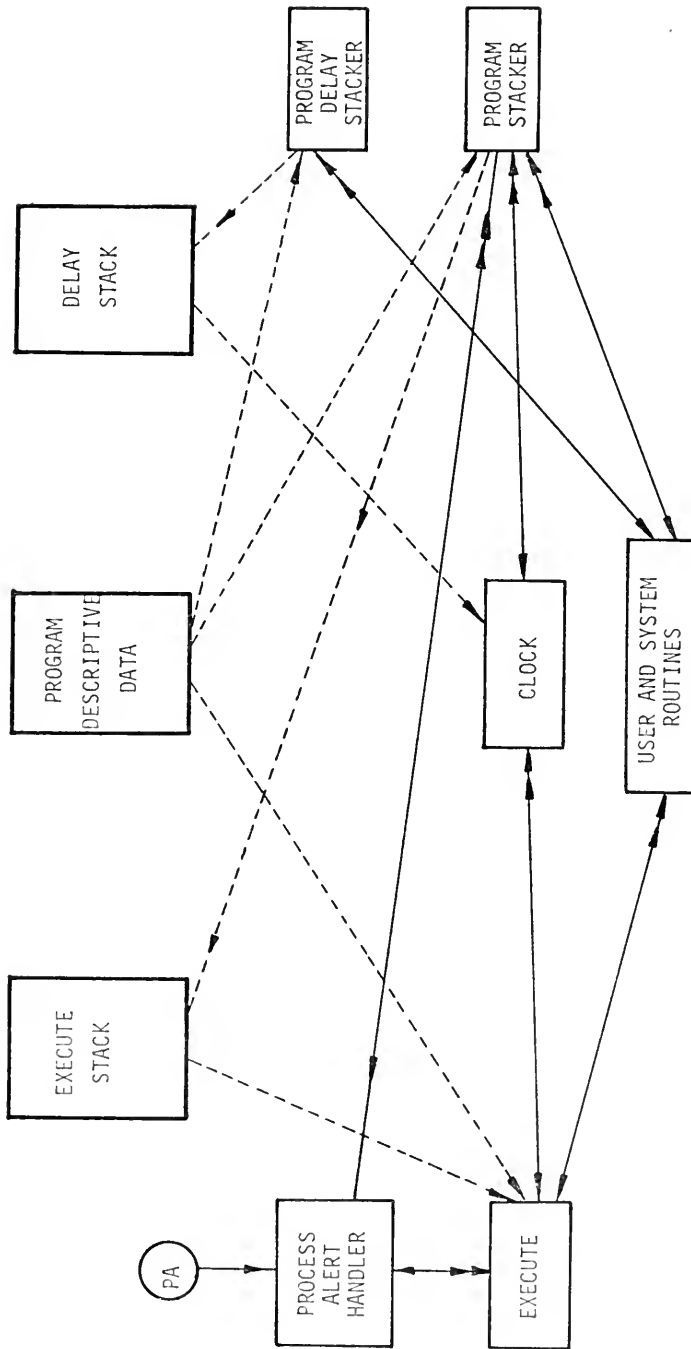


Figure 2.6 Software Setup

subprogram uses to pass control to CLOCK and the user and system subprograms.

The computer costs are extremely low when based mainly on central processing unit (CPU) time. The software utilizes very little CPU time. Typical costs are in the range of \$3-5 an hour provided no elaborate computations are called for in the user programs. However, costs for core residency charges dominate as the basic software package requires around 20,000 words or 80,000 bytes of core.

CHAPTER III

DYNAMIC MODEL AND PARAMETER ESTIMATION

The dynamic modeling of multiple-effect evaporators has been extensively investigated in recent years at the University of Alberta (Andre and Ritter, 1968), (Newell, 1970). In simulation and experimental work high order, linear models have been found to be satisfactory. However, linear models are not realistic when the operating conditions change drastically as in start-up. In the first part of this chapter a nonlinear, first order, lumped parameter model is proposed. The first order and lumped parameter nature of the model was resorted to for two main reasons:

- 1) The model was simple and adequately described the data.
- 2) The model was used to devise an optimal control policy for minimum time start-up. Optimal control theory has been rigorously developed for lumped parameter systems and its extension to distributed systems has not yet been extensively investigated.

In addition, the model presented here takes into account heat transfer dynamics from the viewpoint of film coefficients. Although this leads to complicated algebraic equations, it has the advantage of leading to a better understanding of the heat transfer dynamics. It also gives rise to two constant correction parameters. The necessity for these parameters is due to the uncertain coefficients that are used in the film coefficient equations. The second part of this chapter deals with the estimation of these parameters to fit the experimental data.

III.1 Dynamic Model

The dynamic model is a collection of the material and energy balances for each effect. For a double effect evaporator concentrating a solution with one major solute, there are two material balances (one for the solution and one for the solute) and one energy balance for each effect, giving rise to a total of six dynamic or state equations for the two effects. In addition, there are dynamic equations for the vapor phases and metal but the time constants of these are negligible compared to the six mentioned earlier (Andre, 1968) so that these dynamic equations could be reduced to be algebraic equations. This procedure of setting the derivatives of the equations with small time constants to zero reduces the order of the system. The full model will be presented here. In later chapters, appropriate simplifications will be applied as some of the model states are held fixed (for example, as boiling does or does not take place). A summary of all the assumptions made is presented at the end of the model. Refer to Figure 3.1 for the symbols used for the flows, hold-ups and temperatures. There is also a foldout nomenclature list on page 148.

III.1.1 State Equations

- 1) First and second effect hold-ups, H_1 and H_2 .

$$\frac{dH_1}{dt} = W_{12} - V_{21} - W_{11} - W_{01} \quad (3.1)$$

$$\frac{dH_2}{dt} = W_F - V_{02} - W_{12} \quad (3.2)$$

- 2) First and second effect enthalpies.

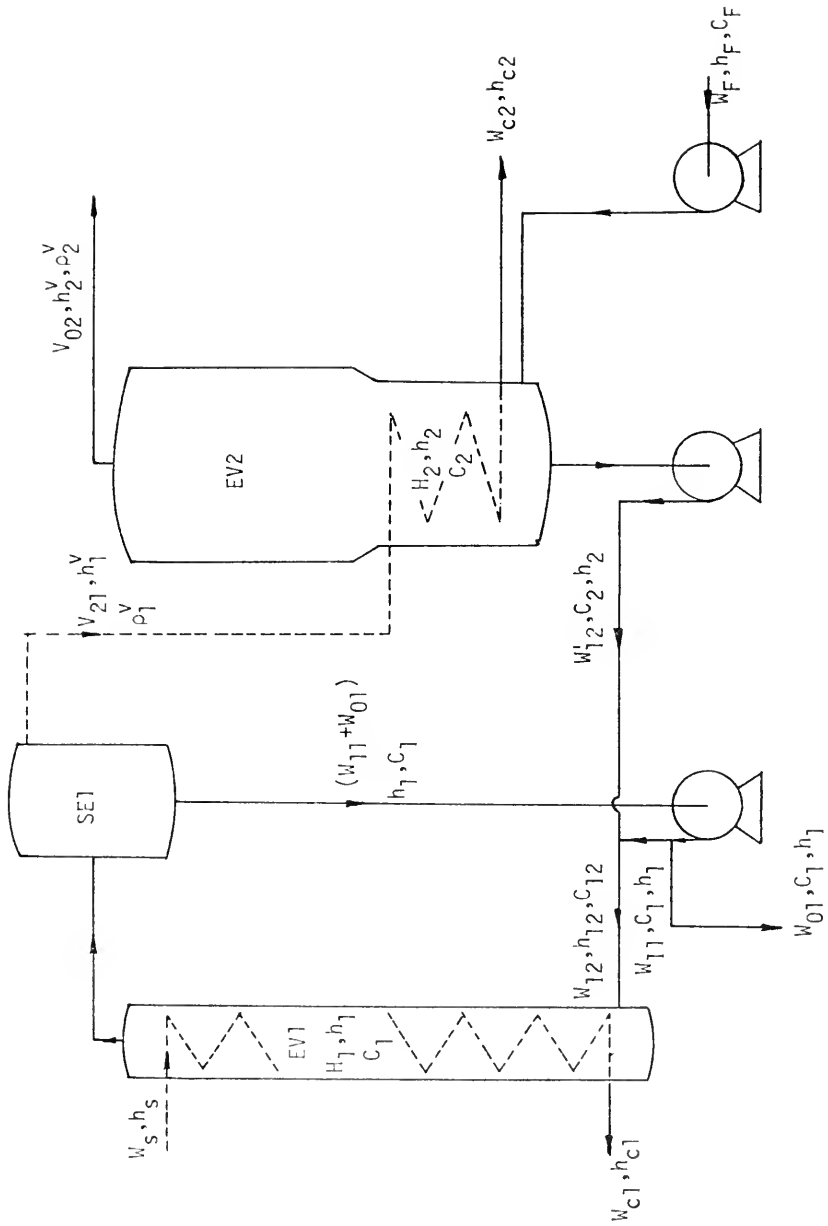


Figure 3.1 Variables for Material and Energy Balances

Since the evaporator was to be used ultimately for the concentration of dilute solutions, it was assumed that there would be no boiling point elevations in the effects. Also, perfect mixing is assumed which would be close to the case for small hold-ups and dilute solutions.

$$\frac{d(H_1 h_1)}{dt} = W_{12} h_{12} - V_{21} h_1^V - (W_{11} + W_{01}) h_1 + Q_1$$

where Q_1 is the heat transferred from the steam. Simplifying this using equation (3.1), we get

$$\frac{dh_1}{dt} = \frac{1}{H_1} [W_{12}(h_{12} - h_1) + V_{21}(h_1 - h_1^V) + Q_1] \quad (3.3)$$

Similarly, an energy balance on the second effect under the same assumptions gives rise to

$$\frac{dh_2}{dt} = \frac{1}{H_2} [W_F(h_F - h_2) + V_{02}(h_2 - h_2^V) + Q_2] \quad (3.4)$$

3) First and second effect solute material balances.

Again, assuming perfect mixing we have for the first effect solute,

$$\frac{d(H_1 C_1)}{dt} = W_{12} C_{12} - (W_{11} + W_{01}) C_1$$

Simplifying this with equation (3.1) we have

$$\frac{dC_1}{dt} = \frac{1}{H_1} [W_{12}(C_{12} - C_1) + V_{21} C_1] \quad (3.5)$$

Similarly, a balance on the second effect solute yields,

$$\frac{dC_2}{dt} = \frac{1}{H_2} [W_F(C_F - C_2) + V_{02} C_2] \quad (3.6)$$

III.1.2 Connection Equations

In addition to the state equations listed above, there are algebraic equations which arise due to the mixing of the two streams between the second and first effects. One energy and two material balances describe the mixing as follows:

$$W_{12} = W_{12}^I + W_{11} \quad (3.7)$$

$$W_{12}C_{12} = W_{12}^IC_{12} + W_{11}C_1 \quad (3.8)$$

$$W_{12}h_{12} = W_{12}^Ih_{12} + W_{11}h_1 \quad (3.9)$$

III.1.3 Heat Transfer Equations

These equations arise in computing the terms Q_1 and Q_2 that arise in the enthalpy equations (3.3) and (3.4).

The heat transfer rate Q_1 is a function of the steam temperature, the first effect temperature, the inside and outside film coefficients, the wall resistance including fouling and the heat transfer area in the first effect. The inside film coefficient is a function of the flow rate through the tubes, the vaporization, the inner wall and bulk temperatures and the entrance temperature. The heat transfer mechanism is initially simple--a combination of the Dittus-Boelter equation for the inside and the Nusselt equation for the condensing steam. However, when boiling takes place two-phase heat transfer occurs because of the vapor formed. The complete boiling mechanism is a topic for further investigation. Approximate correlations were obtained from (Fair, 1960, 1963a, 1963b), (Hughmark, 1969) and more recently from (Tong, 1965). The complete list of equations leading to the determination of Q_1 from the state variables and flow rates is given in Appendix A. Due to the uncertainty in

the empirical equations which predict the inside film coefficients, a parameter θ_1 was introduced in the overall heat transfer equations (A.17) and (A.40). It is assumed that the outside film coefficient is predicted by the Nusselt equation, (A.14) and (A.28) to a reasonably high degree of accuracy as is borne out later by experiment. It is also assumed that the parameter θ_1 has two different values depending upon the heat transfer mechanism in the first effect. This depends upon the stage of start-up as follows:

- 1) $\theta_1 = \theta_{1a}$, when the first effect liquid is being heated. The dominant equation for the inside film coefficient is solely the Dittus-Boelter equation (A.16).
- 2) $\theta_1 = \theta_{1b}$, when the liquid in the first effect is boiling. The inside film coefficient is a combination of many factors including a coefficient due to nucleate boiling (A.38) and a two-phase convective coefficient (A.33).

The equation for the first effect heat transfer rate is given in functional form as:

$$Q_1 = Q_1(T_s, T_1, T_{12}, H_T W_{12}, V_{21}, \theta_1) \quad (3.10)$$

where the temperatures, T , are functions of the enthalpies, h , in the form

$$T = f(h)$$

Equation (3.10) has implicitly used the fact that the overall coefficient, film coefficients and heat transfer area are functions of temperatures (enthalpies) and hold-ups.

Strictly speaking, the equations describing the steam (vapor) temperatures or enthalpies, h_s , h_1 and h_2 are differential equations,

$$V_1^v \frac{d\rho_1^v}{dt} = V_{21} - W_{c2}$$

$$V_1^v \frac{d(\rho_1^v h_1^v)}{dt} = V_{21} h_1^v - W_{c2} h_{c2} - Q_2$$

where V_1^v is the volume of the vapor space in the first effect, the vapor-liquid separator and the tubes of the second effect. Note that two assumptions have been made here--the steam (vapor) is saturated and there is no subcooling of the condensate.

However, it has been shown by Andre and Ritter (1968) that the response rate of the steam enthalpy is negligible compared to that of the hold-up, concentration and enthalpy equations (3.1) to (3.6) in the two effects. The differential equations describing the steam density and temperature are so replaced with the steady state equations

$$V_{21} = W_{c2}$$

and

$$Q_2 = V_{21}(h_1^v - h_{c2})$$

or

$$Q_2 = V_{21}\lambda_1 \quad (3.11)$$

where

$$\lambda_1 = f(T_1)$$

The heat transfer rate in the second effect, Q_2 , is also a function of the film coefficients, wall resistance including fouling, area and temperatures in the second effect. The heat transfer mechanism is purely natural convection. Here again, it is assumed that the Nusselt equation (A.47) is reasonably accurate in predicting the outside film coefficient. The inside film coefficient is predicted by the natural convection equation (A.51) and the overall coefficient (A.52) has an undetermined parameter θ_2 which again can

have two values. One value (θ_{2a}) is for the heating of the liquid and the other (θ_{2b}) is for the boiling of the liquid in the second effect. The functional form for Q_2 is:

$$Q_2 = Q_2(T_1, T_2, T_F, H_2, \theta_2) \quad (3.12)$$

where the temperatures have been determined from the corresponding enthalpies.

Note that there are no liquid flow and vapor flow terms here as the natural convection overall coefficient is not a function of these variables.

A subroutine called HEAT has been written to calculate the heat transfer rates Q_1 and Q_2 from the temperatures, hold-ups and flow rates. It is included in Appendix B. The rates Q_1 and Q_2 are found by using all the heat transfer equations in Appendix A. The inner and outer wall temperatures that figure in the film coefficient calculations are unknown. These temperatures are initially guessed and the film coefficients are calculated. The wall temperatures are adjusted until the equations predict the same heat transfer rate per unit area across the inside and outside films and the wall. With the final wall temperatures, the film coefficients and heat transfer rates are estimated using the appropriate equations depending upon the nature of boiling and the mechanism.

III.1.4 Decision Variables

Equations (3.1) to (3.6) are the differential equations and (3.7) to (3.12) are the algebraic equations describing the dynamic response of the evaporator. After enumerating the number of variables and the number of equations it is found that there are eight more variables than there are equations. These eight decision variables are chosen in a natural way making them manipulative or control

variables. These are:

- 1) feed flow rate, W_F
- 2) feed temperature, T_F
- 3) feed concentration, C_F
- 4) inter-effect flow rate, W'_{12}
- 5) recirculation in first effect, W_{11}
- 6) product flow rate, W_{01}
- 7) steam temperature in first effect, T_s
- 8) total pressure in second effect, P_2

III.1.5 Assumptions

A summary of the assumptions made in writing the model is presented here.

- 1) Time responses of the vapor phase and the tube walls are negligible compared to that of the liquid phase. This results in simpler algebraic equations for the vapor phase and tube walls and also decreases the dimensionality of the model.
- 2) The vapor is saturated and is in equilibrium with the liquid at the same temperature.
- 3) Condensate on the vapor side of the first and second effects is not subcooled and condensate hold-up is negligible.
- 4) Boiling point elevations due to the presence of solute in the two effects is negligible. This is justified in the case of dilute solutions.
- 5) There is perfect mixing in the two effects resulting in lumped parameter concentration and heat transfer equations.
- 6) The heat transfer mechanism in the first effect is single phase convection followed by two-phase convection and nucleate boiling and that in the second effect is natural convection.

- 7) Slug flow is the predominant flow pattern in the first effect when boiling takes place.
- 8) Heat losses are negligible.
- 9) The inaccuracy of the heat transfer rates is due mainly to the uncertainty of the inside film coefficient leading to undetermined parameters to correct for the inside coefficients alone.

III.2 Parameter Estimation

III.2.1 Stochastic versus Deterministic Estimation

An extensive review of parameter estimation techniques in differential equations is available in Nieman et al., (1971). In the deterministic case the simplest and most effective method is a least squares fit. The problem is stated as follows:

Given the state equations

$$\dot{X} = f[X(t), U(t), \theta(t)]$$

where $U(t)$ are the control or manipulative variables and $\theta(t)$ are the parameters to be estimated from experimental data $\hat{Y}(t)$. The observations are related to the states and controls by

$$Y(t) = h[X(t), U(t)]$$

The problem is to determine the parameters $\theta(t)$ such that the model "best fits" the given experimental data $\hat{Y}(t)$.

Assuming that the parameters are constant, as in the case of the evaporator, $\theta(t) = \theta$, the problem can be reduced to a least-squares estimation

$$\text{Min}_{\theta} \sum_{i=1}^N (Y_i - \hat{Y}_i)^2$$

where Y_i is a calculated observation at time t_i ($i=1, \dots, N$) and \hat{Y}_i is

the actual observation.

The alternative to the above deterministic estimation is the problem of stochastic estimation. It seemed that the model would fit the data far better if the parameters Θ were updated as each measurement was made. Further, if the states and measurements were subject to process and measurement noise it would be necessary to estimate the states using a nonlinear or a linearized Kalman filter. It was apparent all along that this would require an appreciable amount of computer time; however, the estimation technique was investigated off-line from an academic viewpoint.

The approach follows that of Padmanabhan (1970). The model is assumed to have process and measurement noise $v(t)$ and $w(t)$

$$\dot{X}(t) = f(X(t), t) + V(t)$$

$$Y(t) = h(X(t), t) + W(t)$$

where $V(t)$ and $W(t)$ are white Gaussian noise sequences with zero mean and covariances $Q(t)$ and $R(t)$ respectively. Note that the control vector $U(t)$ is expressed in terms of the state vector $X(t)$ in the state and observation equations above. To the state equations could be augmented the parameter equations

$$\dot{\Theta}(t) = 0 + V'(t)$$

thus treating the parameters Θ as states.

The problem reduces to estimating the states $\hat{X}(t)$ and the parameters $\hat{\Theta}(t)$ from the experimental data $\hat{Y}(t)$. A recursive estimate $X(t_k/t_k)$ representing an estimate of X at time t_k based on all the data collected until time t_k is given by

$$\frac{d\hat{X}}{dt}(t_k/t_k) = f(\hat{X}(t_k/t_k), t_k) + P(t_k)\hat{Z}(t_k/t_k)$$

with $\hat{X}(0/0) = \mu$ = initial estimate of $X(t_0)$

where $\hat{Z}(t_k/t_k) = h_x^T R(t_k)^{-1} (\hat{Y}(t_k) - h(\hat{X}(t_k/t_k), t_k))$

and the covariance of the estimate $P(t_k)$ satisfies a matrix Ricatti equation

$$\frac{dP}{dt}(t_k) = \hat{f}_x P(t_k) + P(t_k) \hat{f}_x^T + P(t_k) \hat{Z}_x P(t_k) + Q(t_k)$$

with $P(0) = \Omega$ = initial covariance of estimate $X(t_0)$. In the linear case the covariance equation can be integrated off-line and stored for use by the state estimation equations.

To study the effectiveness of the scheme an example was run off-line in which it was assumed that the first and second effect hold-ups and the first effect temperature were constant. It was desired to estimate the second effect temperature and the parameter θ' in the equation

$$\frac{dh_2}{dt} = \frac{1}{H_2} [W_F(h_F - h_2) + \theta' Q_2]$$

while measurements were obtained on the second effect temperature

$$T_2 = f(h_2)$$

The state equation and the Ricatti equations were integrated for eight minutes of real time and this took 100 seconds of IBM 370/165 CPU time. The results for the first four minutes were as follows:

<u>Time</u>	<u>$x_{4,\text{measured}}$</u>	<u>\hat{x}_4</u>	<u>$x_{4,\text{predicted}}$</u>	<u>θ'</u>
30.7	118.36			1.0
31.7	122.63	121.4	118.73	0.08
32.7	124.32	124.1	122.37	0.18
33.7	125.82	126.76	125.05	0.19
34.7	126.62	127.34	126.35	0.17

This example showed that it was not practical to try on-line stochastic estimation for a problem of this nature especially when there are more state variables. Because of this the deterministic least-squares estimate was resorted to and the results obtained were acceptably good.

To account for noisy flow variables a linear filter was used on the flow measurements in the form

$$\hat{u}_i(j) = \alpha \hat{u}_i(j-1) + (1 - \alpha) u_i(j)$$

where $0 < \alpha < 1$.

$\hat{u}_i(j)$ was the filtered value of the flow u_i at t_j . $u_i(j)$ was the measurement of the flow u_i at t_j . $\alpha=0.3$ was a value commonly used.

It was assumed that the temperatures were measured with a high degree of accuracy.

III.2.2 Experimental Work for Determining θ_{1a}

The experimental runs conducted for determining θ_1 and θ_2 were made in two sets, A and B. The runs in set A were made when the first effect was being heated and θ_{1a} was estimated.

Effects 1 and 2 were initially filled to their steady state hold-ups. The product flow, inter-effect flow and feed to the second effect were cut off. The recirculation flow, W_{11} , was held between 10 and 15 gpm (0.6 to 0.9 kg/s). Heating steam was then started to the first effect. Recordings of the inter-effect temperature T_{12} , steam temperature, T_s , and recirculation flow, W_{11} , were made along with the other variables but it was only these variables that figured in the ensuing calculations. Tables 3.1 to 3.5 contain the data from Runs A1 to A5.

TABLE 3.1
EXPERIMENTAL DATA FOR RUN A1

FIRST EFFECT HOLD-UP = 45.00 LBS									
SECOND EFFECT HOLD-UP = 31.00 LBS									
TIME (SECS)	FLOW RATES IN LBS/MIN				TEMPERATURES IN DEG F				
	WF	W12P	W11	W01	TS	T12	T2	TF	
332.84	0.0	0.0	137.66	0.0	241.4	171.2	177.0	177.9	
341.97	0.0	0.0	138.34	0.0	242.3	174.9	176.2	178.0	
351.24	0.0	0.0	132.78	0.0	243.6	178.9	176.7	178.0	
357.09	0.0	0.0	138.16	0.0	244.0	181.1	176.2	178.0	
380.73	0.0	0.0	123.46	0.0	245.2	189.5	175.8	178.0	
394.83	0.0	0.0	136.23	0.0	246.5	192.5	175.8	178.0	
401.87	0.0	0.0	134.27	0.0	246.5	194.7	175.8	178.5	
412.57	0.0	0.0	131.71	0.0	246.6	196.1	175.9	178.1	
419.62	0.0	0.0	129.46	0.0	247.0	197.5	175.5	178.1	
427.85	0.0	0.0	128.30	0.0	247.4	198.3	175.0	178.1	
440.73	0.0	0.0	134.05	0.0	247.8	199.6	175.0	178.6	
446.60	0.0	0.0	132.40	0.0	247.8	200.1	175.5	178.6	
456.45	0.0	0.0	128.53	0.0	248.2	201.8	175.5	178.6	
469.41	0.0	0.0	131.30	0.0	248.2	202.2	174.6	178.6	
482.16	0.0	0.0	134.48	0.0	248.9	203.8	175.3	178.8	
491.60	0.0	0.0	131.99	0.0	248.9	205.5	174.8	178.4	
503.44	0.0	0.0	131.99	0.0	249.3	207.2	174.8	178.8	
521.15	0.0	0.0	125.53	0.0	249.3	209.0	174.4	178.8	
561.77	0.0	0.0	128.58	0.0	238.2	210.7	174.0	178.0	
601.05	0.0	0.0	122.08	0.0	216.7	212.9	173.6	178.5	

TABLE 3.2
EXPERIMENTAL DATA FOR RUN A2

TIME (SECS)	FIRST EFFECT HOLD-UP = 27.00 LBS		SECOND EFFECT HOLD-UP = 26.00 LBS		FLOW RATES IN LBS/MIN		TEMPERATURES IN DEG F			
	WF	W12P	W11	W01	TS	T12	T2	TF		
148.60	0.0	0.0	114.55	0.0	245.5	179.2	167.1	179.2		
154.42	0.0	0.0	109.20	0.0	245.9	181.8	167.1	179.2		
162.66	0.0	0.0	109.72	0.0	246.3	182.7	167.1	178.8		
173.29	0.0	0.0	110.53	0.0	246.8	186.7	167.1	179.2		
182.72	0.0	0.0	108.64	0.0	246.8	189.8	167.1	179.2		
194.55	0.0	0.0	110.16	0.0	247.6	193.3	166.7	179.2		
204.37	0.0	0.0	109.20	0.0	247.4	195.8	167.0	179.1		
215.50	0.0	0.0	105.16	0.0	247.9	197.1	166.6	179.1		
228.51	0.0	0.0	110.64	0.0	248.3	198.8	166.6	179.1		
246.15	0.0	0.0	108.38	0.0	249.1	201.0	166.6	179.1		
259.17	0.0	0.0	105.08	0.0	249.1	202.3	166.1	179.1		
275.49	0.0	0.0	105.81	0.0	248.8	204.5	165.3	178.7		
281.32	0.0	0.0	109.98	0.0	249.2	204.9	165.7	179.1		
293.15	0.0	0.0	103.69	0.0	249.2	206.2	165.3	178.7		
301.38	0.0	0.0	107.56	0.0	249.2	207.1	164.8	178.7		
314.42	0.0	0.0	101.04	0.0	249.6	207.9	165.3	178.2		
320.25	0.0	0.0	105.54	0.0	249.2	208.4	165.3	178.7		
349.63	0.0	0.0	105.55	0.0	249.6	209.6	164.8	179.1		
360.28	0.0	0.0	102.97	0.0	250.0	210.5	164.8	178.7		
380.50	0.0	0.0	104.65	0.0	249.6	210.9	164.4	179.1		

TABLE 3.3
EXPERIMENTAL DATA FOR RUN A3

FIRST EFFECT HOLD-UP = 32.00 LBS				TEMPERATURES			
SECOND EFFECT HOLD-UP= 26.00 LBS				IN DEG F			
TIME	FLOW RATES IN LBS/MIN			TS	T12	T2	TF
(SECS)	WF	W12P	W11	W01			
176.62	0.0	0.0	126.82	0.0	243.9	178.4	177.9
182.46	0.0	0.0	121.54	0.0	244.8	181.0	177.9
188.29	0.0	0.0	123.40	0.0	246.0	185.0	177.9
196.36	0.0	0.0	118.04	0.0	246.8	187.2	177.9
202.19	0.0	0.0	120.06	0.0	246.8	189.0	177.9
211.31	0.0	0.0	111.55	0.0	246.7	190.6	177.8
217.14	0.0	0.0	113.86	0.0	247.5	191.5	177.8
222.97	0.0	0.0	116.31	0.0	247.5	194.5	177.8
230.00	0.0	0.0	118.22	0.0	247.9	197.1	177.8
235.83	0.0	0.0	113.93	0.0	248.3	198.5	177.8
242.90	0.0	0.0	114.08	0.0	248.3	200.2	177.4
253.54	0.0	0.0	118.57	0.0	248.8	201.5	177.4
271.18	0.0	0.0	111.81	0.0	248.8	204.1	177.4
290.94	0.0	0.0	116.74	0.0	248.8	205.9	177.0
299.18	0.0	0.0	116.81	0.0	249.2	207.1	177.0
305.02	0.0	0.0	117.27	0.0	249.2	207.6	176.5
319.08	0.0	0.0	112.58	0.0	249.2	208.4	176.5
327.32	0.0	0.0	118.78	0.0	249.2	209.3	176.5
349.69	0.0	0.0	111.06	0.0	238.2	210.7	176.2
361.52	0.0	0.0	113.75	0.0	219.2	211.1	176.2

TABLE 3.4
EXPERIMENTAL DATA FOR RUN A4

FIRST EFFECT HOLD-UP = 50.00 LBS									
SECOND EFFECT HOLD-UP = 28.00 LBS									
TIME (SECS)	WF	W12P	FLOW RATES IN LBS/MIN		TEMPERATURES		IN DEG F		TF
			W11	W01	TS	T12	T2		
210.67	0.0	0.0	118.32	0.0	245.4	184.4	180.4	185.3	
216.52	0.0	0.0	124.80	0.0	245.4	186.6	180.4	135.3	
222.35	0.0	0.0	119.47	0.0	245.8	188.3	180.4	135.3	
237.33	0.0	0.0	125.43	0.0	245.8	191.0	179.5	134.8	
243.16	0.0	0.0	123.80	0.0	246.7	192.3	179.5	184.8	
254.86	0.0	0.0	121.64	0.0	246.7	194.9	179.5	185.3	
261.90	0.0	0.0	122.79	0.0	247.1	195.8	179.1	135.3	
272.54	0.0	0.0	125.26	0.0	247.5	197.1	179.1	134.8	
280.79	0.0	0.0	123.80	0.0	247.9	198.4	179.1	134.8	
286.63	0.0	0.0	121.23	0.0	247.9	199.3	179.1	134.8	
300.61	0.0	0.0	122.46	0.0	248.2	200.5	179.0	134.7	
312.57	0.0	0.0	122.39	0.0	248.2	201.3	178.5	135.2	
327.85	0.0	0.0	125.00	0.0	248.6	203.1	178.1	135.2	
336.09	0.0	0.0	119.59	0.0	248.6	204.4	178.1	135.2	
346.81	0.0	0.0	117.76	0.0	249.0	205.2	178.1	135.2	
357.46	0.0	0.0	122.15	0.0	249.0	206.1	178.1	135.6	
366.60	0.0	0.0	113.02	0.0	249.1	207.4	178.1	135.6	
379.47	0.0	0.0	123.80	0.0	249.1	208.3	178.1	135.6	
390.11	0.0	0.0	123.03	0.0	249.5	209.1	178.1	135.2	
405.58	0.0	0.0	119.06	0.0	249.5	210.0	178.6	135.2	

TABLE 3.5

EXPERIMENTAL DATA FOR RUN A5

TIME (SECS)	FIRST EFFECT HOLD-UP = 27.00 LBS		SECOND EFFECT HOLD-UP= 33.00 LBS		FLOW RATES IN LBS/MIN		TEMPERATURES IN DEG F			
	WF	W12P	W11	W01	TS	T12	T2	TF		
192.54	0.0	0.0	102.41	0.0	243.8	174.7	172.0	172.0		
198.37	0.0	0.0	114.12	0.0	244.7	177.8	172.0	171.6		
205.40	0.0	0.0	104.33	0.0	245.5	181.8	172.0	171.2		
211.27	0.0	0.0	105.55	0.0	245.9	184.0	172.0	171.2		
226.24	0.0	0.0	107.03	0.0	246.7	188.0	172.0	171.6		
232.08	0.0	0.0	105.63	0.0	247.1	190.2	172.5	171.6		
237.94	0.0	0.0	107.96	0.0	247.1	191.9	172.0	171.6		
243.76	0.0	0.0	105.04	0.0	247.5	194.1	172.0	172.0		
249.61	0.0	0.0	108.46	0.0	247.5	196.3	172.0	171.6		
261.26	0.0	0.0	110.87	0.0	247.5	198.9	172.0	171.6		
272.94	0.0	0.0	106.10	0.0	248.4	200.6	172.0	171.6		
278.77	0.0	0.0	110.87	0.0	248.4	201.5	172.0	171.6		
289.93	0.0	0.0	109.18	0.0	248.7	203.1	171.9	171.0		
295.80	0.0	0.0	106.88	0.0	249.1	204.0	171.9	171.0		
313.33	0.0	0.0	101.55	0.0	249.1	205.7	171.5	171.5		
325.00	0.0	0.0	100.07	0.0	249.1	207.0	171.5	171.5		
332.03	0.0	0.0	110.95	0.0	249.1	208.3	171.5	171.5		
343.69	0.0	0.0	97.90	0.0	249.5	208.8	171.0	171.5		
349.52	0.0	0.0	102.53	0.0	249.5	209.2	171.5	171.5		
359.84	0.0	0.0	104.45	0.0	250.1	210.2	171.2	171.7		

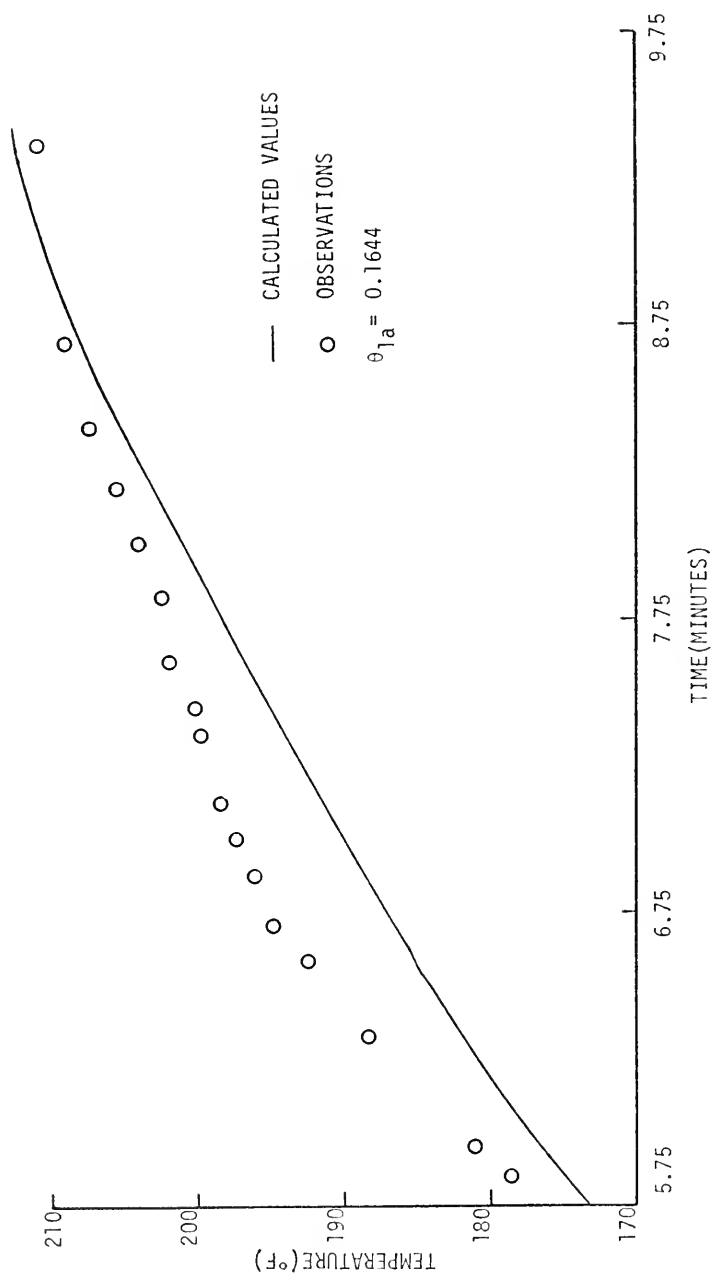
III.2.3 Calculations and Results for θ_{1a}

A computer program was written to estimate θ_{1a} from the data collected in Runs A1 to A5. The least-squares program used was subroutine RMINSQ (Westerberg, 1969). This program was based on a program coded by M. J. D. Powell and described in Powell (1964). This routine has the capability of performing a least-squares search over several functions in several variables. The search routine does not require evaluation of derivatives.

The equation describing the enthalpy rise in the first effect is equation (3.3).

$$\frac{dh_1}{dt} = \frac{1}{H_1} [W_{12}(h_{12} - h_1) + V_{21}(h_1 - h_1^v) + Q_1] \quad (3.3)$$

Runs A1 to A5 were conducted when the first effect was not boiling and with constant hold-ups in the first and second effects. Thus, the liquid entering the first effect was only the recirculated liquid, $W_{12} = W_{11}$ and the vaporization was zero, $V_{21} = 0$. The observation $T_{12} = T_1$ since all the liquid entering the first effect was recirculated. For every value of θ_{1a} which subroutine RMINSQ searched over, equation (3.3) was integrated from the initial to the final time for each run. Ten functions of the form $(T_{1,calc} - T_{1,observed})^2$ over the time span were minimized by RMINSQ. The functions toward the end of the time interval were weighted one hundred times more than those at the start. This was because the final time at which the first effect liquid started to boil was more important from the point of view of possible switching times in the control variables at this point in time. The results of the minimization are tabulated in Tables 3.6 to 3.10 and these values are plotted in Figures 3.2 to 3.6. It can be seen that the

Figure 3.2 Calculated versus Observed Values of T_1 for Run A1

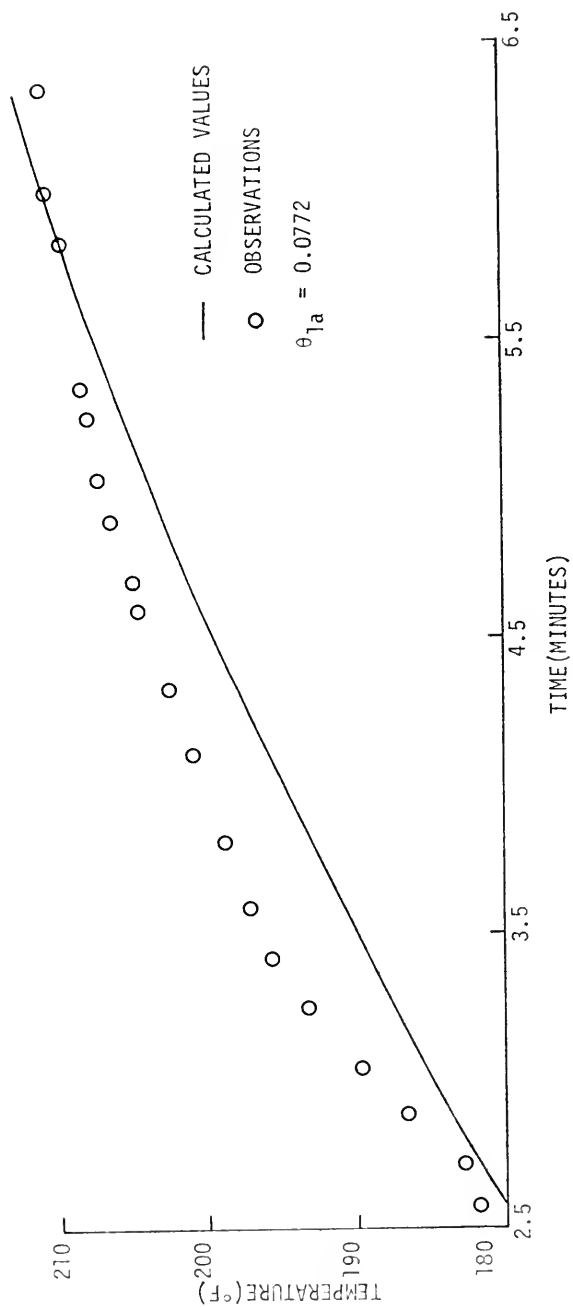


Figure 3.3 Calculated versus Observed Values of T_1 for Run A2

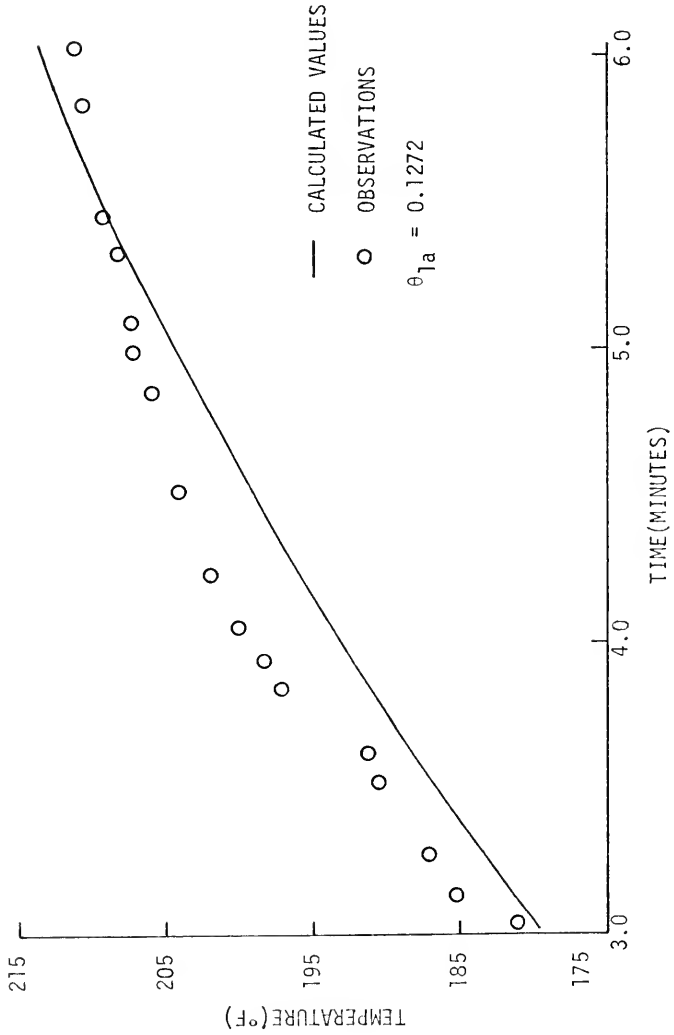


Figure 3.4 Calculated versus Observed Values of T_1 for Run A3

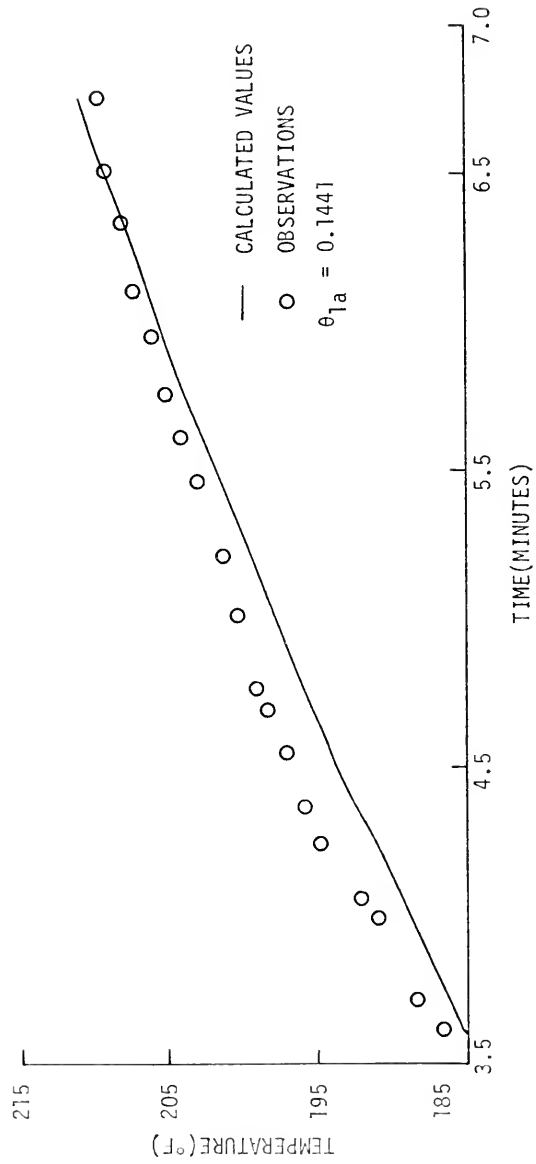


Figure 3.5 Calculated versus Observed Values of T_1 for Run A4

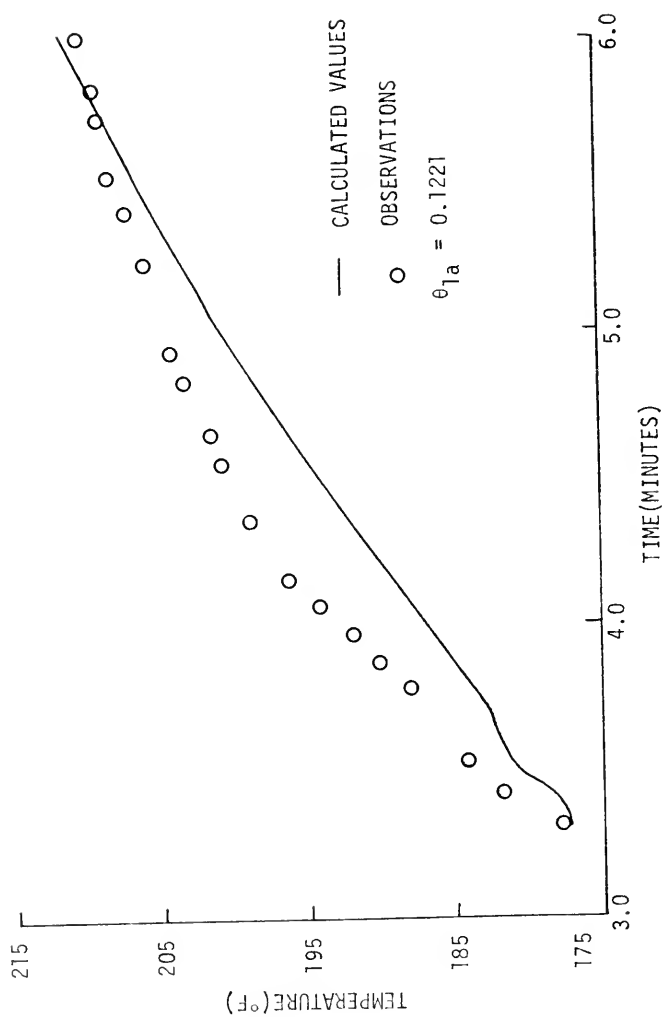


Figure 3.6 Calculated versus Observed Values of T_1 for Run A5

TABLE 3.6
CALCULATED VERSUS OBSERVED VALUES OF T_1 FOR RUN A1

TIME (MINS)	ESTIMATED THETA = 0.1644		T1 IN DEG F OBSERVED
	TEMPERATURES CALCULATED		
5.700	173.37	174.90	178.90
5.854	175.53	176.86	181.12
5.951	182.07	185.04	189.48
6.345	186.49	188.65	192.55
6.580	190.04	191.62	194.73
6.658	194.03	195.10	196.15
6.876	196.86	199.13	197.46
6.994	201.26	202.23	198.33
7.131	202.85	204.75	199.63
7.346	207.46	208.96	200.06
7.443	213.02	216.55	201.80
7.607			202.23
7.823			203.79
8.036			205.52
8.193			207.24
8.391			208.96
8.686			210.74
9.363			212.88
10.018			

TABLE 3.7

CALCULATED VERSUS OBSERVED VALUES OF T_1 FOR RUN A2				
TIME (MINS)	ESTIMATED $\theta_{TA1A} = 0.0772$		TEMPERATURES T_1 IN DEG F	
	CALCULATED		OBSERVED	OBSERVED
2.574	180.29		181.84	
2.711	181.83		182.73	
2.888	183.77		186.70	
3.045	185.44		189.77	
3.243	187.49		193.27	
3.415	189.23		195.76	
3.592	190.98		197.07	
3.809	193.06		198.81	
4.103	195.80		200.98	
4.320	197.77		202.28	
4.592	200.16		204.50	
4.689	200.98		204.94	
4.886	202.61		206.23	
5.023	203.72		207.09	
5.240	205.41		207.95	
5.337	206.15		208.38	
5.827	209.66		209.65	
6.005	210.87		210.51	
6.342	213.08		210.94	

TABLE 3.8

ESTIMATED THETA = 0.1272			
TIME (MINS)	TEMPERATURES T1 CALCULATED	T1 IN DEG F OBSERVED	
3.041	179.87	181.05	
3.138	181.32	185.02	
3.273	183.27	187.22	
3.370	184.64	188.98	
3.522	186.73	190.61	
3.619	188.04	191.48	
3.716	189.32	194.54	
3.833	190.83	197.15	
3.931	192.06	198.46	
4.048	193.53	200.19	
4.226	195.70	201.49	
4.520	199.13	204.09	
4.849	202.74	205.86	
4.986	204.18	207.15	
5.084	205.19	207.58	
5.318	207.54	208.44	
5.455	208.86	209.30	
5.828	212.25	210.69	
6.025	213.54	211.12	

TABLE 3.9
CALCULATED VERSUS OBSERVED VALUES OF T_1 FOR RUN A4

TIME (MINS)	ESTIMATED $\theta_{E1A1A} = 0.1441$ TEMPERATURES T_1 IN DEG F	
	CALCULATED	OBSERVED
3.609	185.31	186.57
3.706	186.25	188.33
3.955	188.60	191.00
4.353	189.50	192.31
4.053	189.50	192.31
4.248	191.29	194.93
4.365	192.34	195.80
4.542	193.92	197.11
4.680	195.12	198.41
4.777	195.97	199.28
5.010	197.94	200.48
5.209	199.58	201.35
5.464	201.60	203.08
5.601	202.67	204.38
5.780	204.04	205.24
5.958	205.36	206.10
6.110	206.46	207.41
6.324	207.97	208.27
6.502	209.18	209.13
6.760	210.89	209.99

TABLE 3.10
CALCULATED VERSUS OBSERVED VALUES OF T_1 FOR RUN A5

TIME (MINS)	ESTIMATED $\theta_{T_1} = 0.1221$ TEMPERATURES T_1 IN DEG F	
	CALCULATED	OBSERVED
3.306	176.32	177.84
3.423	178.21	181.83
3.521	179.76	184.03
3.771	183.62	188.00
3.868	185.09	190.19
3.966	186.43	191.94
4.063	187.96	194.13
4.160	189.36	196.30
4.354	192.11	198.91
4.549	194.77	200.65
4.646	196.07	201.52
4.832	198.49	203.14
4.930	199.73	204.01
5.222	203.27	205.73
5.417	205.48	207.03
5.534	206.26	208.32
5.534	206.76	208.32
5.728	208.81	208.75
5.825	209.80	209.18
5.997	211.51	210.20

calculated values are indeed least-squares values (with the weighting indicated) which converge on the observed values toward the end times.

On the basis of these results a value for θ_{1a} was taken as 0.1395.

III.2.4 Experimental Work for Determining θ_{1b} and θ_2

The runs in Set B were made with the first effect boiling and the second effect initially in the heating and later in the boiling stage. Three parameters were actually estimated from the data for each run. θ_{1b} was a parameter for the boiling of the liquid in the first effect, θ_{2a} was a similar parameter for the heating of the liquid in the second effect and θ_{2b} was a parameter for the boiling of the liquid in the second effect. Note that θ_{2b} is, in effect, the revised value of θ_{2a} when there is boiling in the second effect.

The experimental procedure consisted of bringing the hold-ups in the two effects to their steady state values. The controllers in the Transducing and Controlling cabinet (Chapter II) were used to maintain the hold-ups constant. One controller was used on the first effect and another on the second. The pneumatic input signal to the first controller was the output from the DP cell which measured the height (hold-up) in the first effect. The pneumatic output from this controller was directed to valve CV2 (Figure 2.2). Thus analog control of the first effect hold-up was achieved by manipulating flow W_{12} which is the liquid input stream to the first effect from the second effect. Likewise, analog control of the second effect hold-up was achieved by manipulating flow W_F which is the input stream to the second effect.

When the hold-ups were about constant, steam was let in to the first effect for heating and a vacuum of around ten inches' mercury

(0.675 bar) was maintained in the second effect. Data were recorded when the first effect temperature was near boiling. The results for three runs B1, B2 and B3 are shown in Tables 3.11, 3.12 and 3.13 respectively.

III.2.5 Calculations and Results for θ_{1b} and θ_2

A similar least-squares search using subroutine RMINSQ was used to estimate θ_{1b} , θ_{2a} , and θ_{2b} from the data obtained in Runs B1 to B3. The function evaluations for RMINSQ entailed integration of all four of the differential equations (3.1) to (3.4) as neither the hold-ups nor the temperatures were held constant. Whenever the second effect temperature corresponded to the temperature of boiling in the second effect (which was found from the pressure observed) parameter θ_{2b} was used instead of θ_{2a} . The calculated temperature of the first effect solution T_1 and that of the second effect solution T_2 were obtained from the integration of the state equations (3.1) and (3.3) respectively. The criteria for minimization were the functions

$$f(1) = \sum_{i=1}^{10} (T_{1,calc} - T_{1,observed})^2 \quad (3.3)$$

$$f(2) = \sum_{i=1}^{10} (T_{2,calc} - T_{2,observed})^2 \quad (3.4)$$

Ten functions of each type were evaluated in the time span of each run resulting in a total of 20 functions for the evaluation of θ_{1b} , θ_{2a} and θ_{2b} .

The correspondence between the observed and calculated values is shown in Tables 3.14 to 3.16, while Figures 3.7 to 3.9 are plots of these values. The minimization took an average of three minutes CPU time on the IBM 370 for each run. This was mainly due to the three-dimensional

TABLE 3.11

EXPERIMENTAL DATA FOR RUN B1

PRODUCT FLOW RATE, W01 = 0. LBS/MIN										TEMPERATURES			
TIME (SECS)	WF	FLCW RATES IN		VAC PRESS	HOLD-UPS		INS HG	P2	W11	IN DEG F			
		LBS/MIN	LBS/MIN		INS	H2				TS	T12	T2	TF
293.8	0.0	0.0	0.0	82.81	9.96	36.72	31.51	249.40	210.79	172.74	174.52		
322.3	0.0	0.0	0.0	79.23	9.59	36.26	30.49	249.81	212.50	172.74	174.97		
353.4	0.0	0.0	0.0	75.77	9.54	36.19	28.59	245.35	212.87	172.67	174.89		
377.2	0.0	0.0	0.0	77.62	9.50	36.29	27.18	248.94	212.87	192.54	175.34		
405.7	0.0	0.0	3.19	78.34	9.58	35.69	27.06	248.11	211.16	194.29	175.78		
433.8	8.53	5.40	81.60	77.11	9.74	36.29	28.71	247.24	210.24	191.60	176.16		
461.1	6.73	4.14	77.11	9.91	36.52	29.65	246.83	209.81	189.41	176.60			
491.9	5.27	0.0	83.71	9.87	36.90	30.58	247.31	210.75	190.37	177.14			
516.8	3.04	0.0	78.55	9.81	37.03	30.88	247.72	212.03	192.12	177.58			
546.3	0.0	0.0	79.65	9.84	37.07	31.17	248.54	212.89	192.56	178.02			
573.9	0.0	0.0	87.54	10.01	36.91	32.66	248.48	214.09	192.47	178.38			
601.3	0.0	0.0	81.15	10.04	37.25	32.75	248.48	214.52	192.47	178.82			
631.8	0.0	0.0	79.60	9.77	36.56	31.00	248.95	215.45	193.00	178.91			
643.5	0.0	0.0	83.80	9.68	36.81	30.88	248.95	215.45	193.43	178.91			
656.3	0.0	0.0	82.26	9.74	36.75	30.66	248.95	215.45	193.43	179.35			
670.4	0.0	0.0	82.26	9.74	36.47	30.58	248.95	215.88	193.87	179.35			
684.6	0.0	0.0	86.19	9.71	36.59	30.71	248.95	216.30	194.31	179.35			
702.2	0.0	0.0	85.92	9.55	37.20	32.36	248.90	216.24	194.68	179.28			
712.8	0.0	0.0	78.49	9.94	36.98	32.31	248.90	215.82	194.24	179.28			
727.1	0.0	0.0	81.10	9.92	37.10	32.66	248.90	216.24	195.11	179.28			
741.3	0.0	0.0	80.83	9.50	36.96	32.01	248.49	216.24	194.68	179.73			
754.4	0.0	0.0	82.24	9.84	36.68	31.79	248.90	216.24	195.11	180.17			
772.4	0.0	0.0	85.43	9.84	36.95	32.36	248.87	216.22	195.09	179.71			
783.1	0.0	0.0	81.51	9.86	36.81	32.01	248.87	215.79	195.53	179.71			
796.1	0.0	0.0	83.77	9.79	36.85	31.97	248.87	215.79	195.09	179.71			
810.4	0.0	0.0	79.67	9.90	36.90	32.75	248.87	215.79	195.09	179.71			

TABLE 3.12

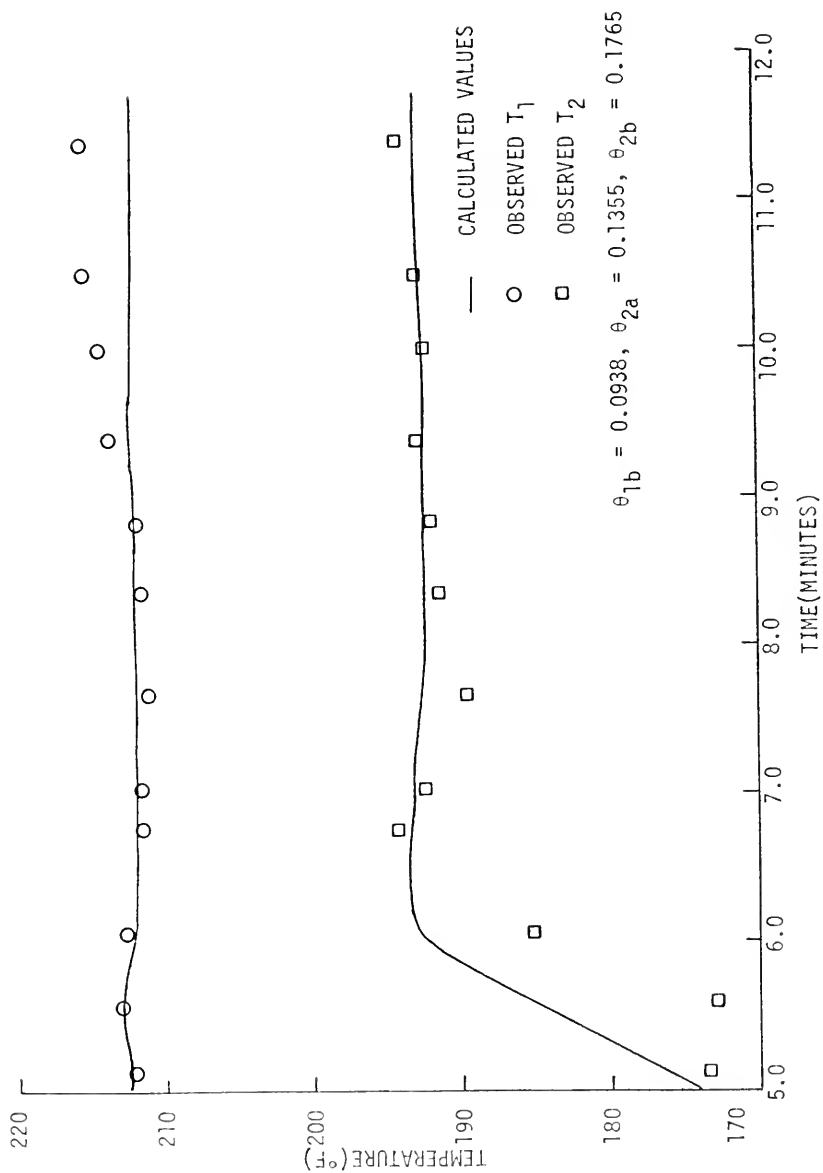
EXPERIMENTAL DATA FOR RUN B2

PRODUCT FLOW RATE, WOI = 0. LBS/MIN									
FLOW RATES IN		VAC PRESS		HOLD-UPS		TEMPERATURES			
TIME	WF	W12P	W11	P2	H1	H2	TS	IN DEG F	TF
(SECS)									
531.9	0.0	0.0	48.22	10.49	35.49	31.81	251.35	211.11	185.47
562.5	0.0	0.0	51.94	10.32	35.43	30.58	251.35	214.10	185.03
587.4	0.0	0.0	51.06	10.01	35.37	29.60	251.35	214.96	190.30
615.0	2.13	0.0	49.15	9.68	35.42	28.42	251.35	213.68	190.74
642.3	9.91	7.41	48.40	9.95	34.95	27.35	249.76	211.15	189.02
656.6	9.49	9.85	49.96	10.00	34.67	28.26	249.35	212.01	188.59
671.3	9.06	9.20	52.49	9.96	34.97	28.84	249.35	211.58	188.15
684.9	7.31	1.40	51.55	9.97	35.02	29.46	249.76	212.01	187.71
702.5	4.78	4.46	56.16	10.22	35.78	31.47	249.76	212.01	187.27
715.4	3.56	0.0	54.51	10.26	35.69	31.77	250.17	212.86	187.71
727.1	0.0	0.0	55.71	10.28	35.70	32.11	250.58	213.72	187.27
742.4	0.0	0.0	55.04	10.26	35.78	32.74	250.99	214.57	187.27
756.6	0.0	0.0	55.56	10.05	35.69	32.70	250.99	215.85	188.15
771.8	0.0	0.0	55.41	9.76	35.60	32.45	250.98	216.69	191.20
782.8	0.0	0.0	50.48	9.60	35.70	32.40	251.39	217.12	192.51
797.1	0.0	0.0	52.26	9.44	35.69	32.74	251.39	217.12	193.39
811.3	0.0	0.0	55.04	9.32	35.70	32.45	251.39	217.54	194.26
825.6	0.0	0.0	55.04	9.11	35.33	32.40	251.80	217.97	194.70
842.0	0.0	0.0	62.05	8.80	35.70	32.11	251.89	218.50	195.24
852.8	0.0	0.0	56.74	8.52	35.67	32.49	251.49	219.35	194.80
867.0	0.0	0.0	58.68	8.15	35.78	32.11	251.89	220.20	194.80
881.4	0.0	0.0	60.28	7.91	35.52	32.45	251.89	221.05	195.24
895.6	0.0	0.0	57.76	7.61	35.78	32.40	252.30	221.05	195.24
914.0	0.0	0.0	54.49	7.42	35.86	33.57	252.22	221.81	195.59
923.4	0.0	0.0	52.42	7.22	35.91	33.27	252.22	220.96	196.02

TABLE 3.13

EXPERIMENTAL DATA FOR RUN B3

PRODUCT FLOW RATE, WOI = 0. LBS/MIN										TEMPERATURES			
TIME		FLW RATES IN		VAC PRESS		HOLD-UPS		INS HG		IN DEG F		T2	
(SECS)	WF	W12P	W11	P2	H1	H2	TS	TS	TS	TS	TS	TS	TS
627.0	4.79	5.83	75.68	9.75	39.23	31.18	257.88	210.23	184.99	173.91			
654.2	5.20	4.59	71.89	9.62	39.15	30.97	257.88	210.66	183.67	173.47			
686.1	6.08	6.94	70.49	9.25	38.43	29.14	258.19	211.84	183.56	173.36			
711.3	7.41	7.26	68.34	9.39	38.64	29.77	257.78	211.84	184.00	173.80			
739.7	8.37	8.03	73.87	9.44	39.08	29.73	258.19	212.27	184.00	173.80			
767.9	10.64	8.47	72.50	9.26	37.77	27.89	257.92	211.97	184.13	174.38			
795.1	10.61	10.54	68.18	9.43	38.18	28.51	257.92	211.54	185.01	174.38			
827.0	8.40	8.09	75.04	9.84	39.12	30.49	258.19	212.27	184.44	174.69			
852.0	8.23	11.52	72.63	9.78	38.89	30.16	258.19	213.12	186.65	175.14			
880.5	7.93	6.99	69.60	9.85	38.23	30.37	258.19	212.69	187.97	176.03			
908.6	7.19	7.05	69.30	9.76	37.12	30.20	258.11	213.04	190.51	175.94			
936.0	7.58	8.29	66.18	9.79	37.70	30.07	257.70	210.89	191.39	176.38			
967.8	7.69	6.51	70.57	10.00	37.41	31.18	258.05	213.42	191.79	177.68			
978.4	7.90	4.88	68.19	10.03	37.17	31.62	258.05	212.99	191.79	177.68			
992.7	5.55	0.0	69.42	10.01	37.87	32.14	258.05	213.85	191.35	177.68			
1007.5	3.47	1.94	74.44	10.02	37.38	32.44	258.87	215.13	193.10	177.68			
1023.8	2.60	4.91	68.81	9.96	37.80	32.18	258.46	216.41	193.97	177.68			
1038.9	1.72	5.86	71.17	9.96	38.13	32.14	258.14	216.05	193.60	177.29			
1049.4	0.0	5.73	72.00	9.99	37.53	32.10	258.55	216.05	194.47	177.74			
1063.4	0.0	6.22	73.98	9.96	37.58	31.92	258.55	216.05	194.47	177.74			
1078.7	0.0	6.53	73.58	9.90	37.66	31.79	258.55	216.48	194.47	178.18			
1092.9	0.0	7.72	70.51	9.83	37.00	31.18	258.55	216.91	194.47	178.18			
1109.1	0.92	3.10	72.57	9.58	37.34	29.77	258.51	217.31	194.01	178.16			
1119.5	4.32	0.0	67.85	9.61	36.84	29.47	258.92	216.88	193.57	178.16			
1133.6	3.77	4.46	67.85	9.66	36.93	29.77	258.92	216.88	193.14	178.61			

Figure 3.7 Calculated versus Observed Values of T_1 and T_2 for Run B1

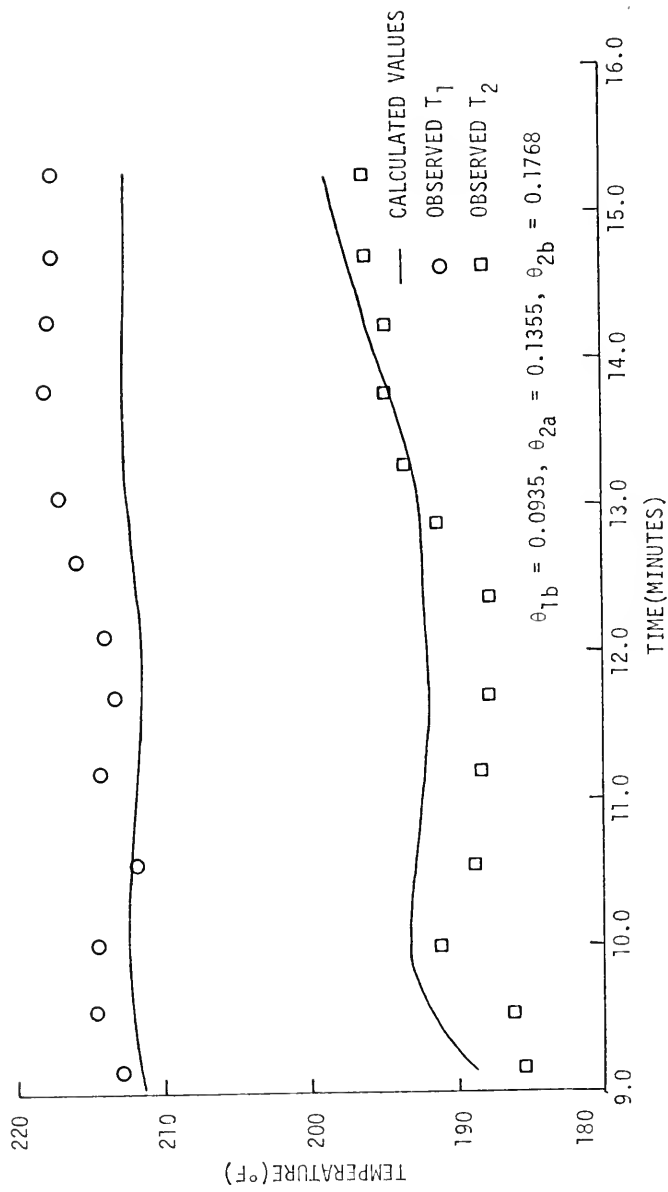


Figure 3.8 Calculated versus Observed Values of T_1 and T_2 for Run B2

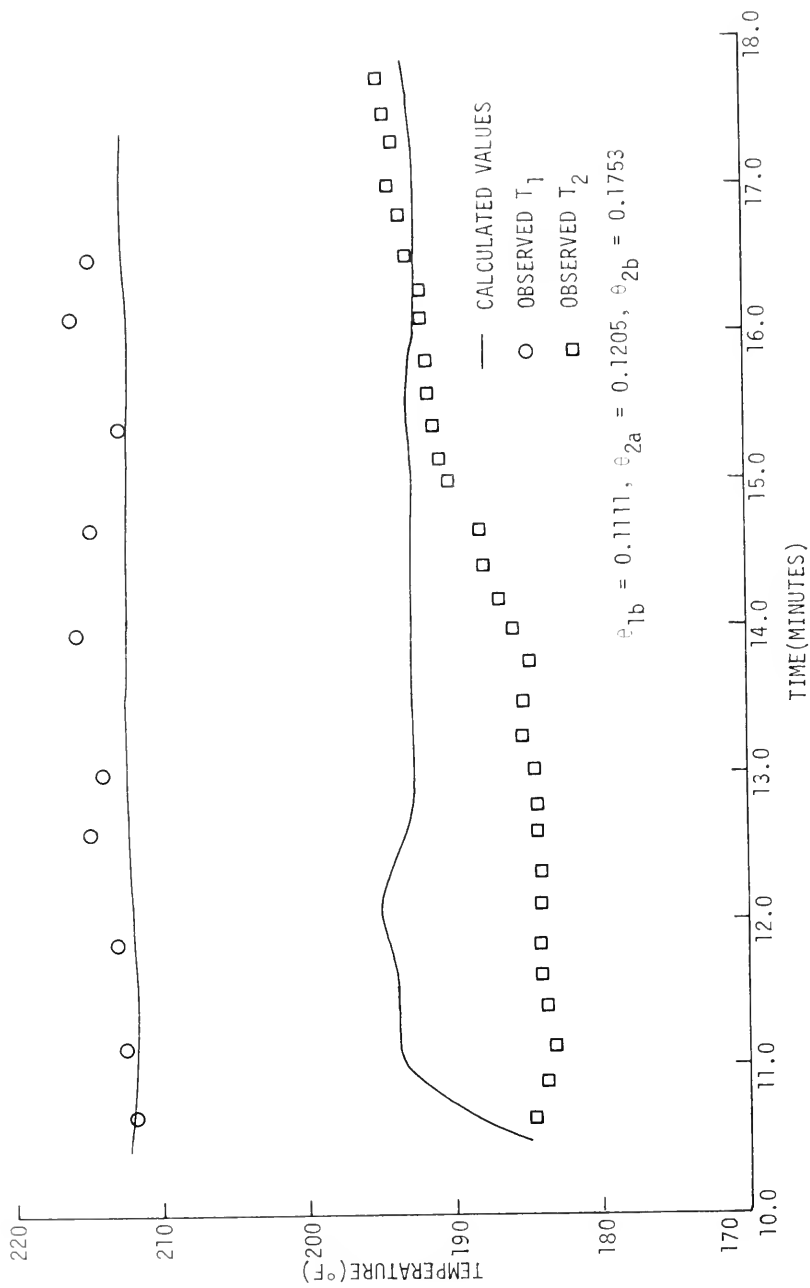


Figure 3.9 Calculated versus Observed Values of T_1 and T_2 for Run 83

TABLE 3.14

CALCULATED VERSUS OBSERVED VALUES OF T_1 AND T_2 FOR RUN B1

TIME (MINS)	THETA18 = 0.0938		THETA2A = 0.1355		THETA2B = 0.1765	
	TEMPERATURE T1 IN DEG F		TEMPERATURE T2 IN DEG F		TEMPERATURE T2 IN DEG F	
	OBSERVED	CALCULATED	OBSERVED	CALCULATED	OBSERVED	CALCULATED
4.8959	210.75	210.79	172.76	172.76	172.74	172.74
5.0209	211.63	212.59	173.13	173.13	174.26	174.26
6.1459	212.56	212.19	185.17	185.17	193.46	193.46
7.3803	210.25	211.64	191.48	191.48	192.69	192.69
8.6147	212.16	212.17	192.22	192.22	192.78	192.78
9.8569	213.67	212.06	192.46	192.46	192.60	192.60
11.0991	214.87	212.07	193.57	193.57	193.08	193.08
12.3413	215.54	212.03	192.38	192.38	193.07	193.07

TABLE 3.15
CALCULATED VERSUS OBSERVED VALUES OF T_1 AND T_2 FOR RUN B2

TIME (MINS)	THETA1B = 0.0935		THETA2A = 0.1355		THETA2B = 0.1768	
	TEMPERATURE T1 IN DEG F		TEMPERATURE T2 IN DEG F		TEMPERATURE T2 IN DEG F	
	OBSERVED	CALCULATED	OBSERVED	CALCULATED	OBSERVED	CALCULATED
9.1410	212.82	211.69	185.03	188.90	185.03	188.90
9.5550	214.53	212.14	186.35	191.98	186.35	191.98
10.0080	214.53	212.21	191.18	193.20	191.18	193.20
10.5460	211.95	212.00	189.02	192.88	189.02	192.88
10.9440	214.28	211.79	188.59	192.19	188.59	192.19
11.4150	212.94	211.50	187.71	192.02	187.71	192.02
11.9230	213.18	211.39	187.71	192.20	187.71	192.20
12.3730	214.60	211.65	187.27	192.16	187.27	192.16
12.8630	216.69	212.01	191.20	192.69	191.20	192.69
13.2840	217.22	212.29	193.39	193.55	193.39	193.55
13.7600	217.94	212.50	194.70	194.53	194.70	194.53
14.2130	219.35	212.48	194.80	195.80	194.80	195.80
14.6900	221.05	212.27	195.24	197.40	195.24	197.40
15.2330	221.81	212.38	195.59	198.72	195.59	198.72
15.6250	220.54	212.02	196.89	198.98	196.89	198.98

TABLE 3.16

CALCULATED VERSUS OBSERVED VALUES OF T_1 AND T_2 FOR RUN B3

TIME (MINS)	THETA1B = 0.1111		THETA2A = 0.1205		THETA2B = 0.1753	
	TEMPERATURE OBSERVED	T1 IN DEG F CALCULATED	TEMPERATURE OBSERVED	T2 IN DEG F CALCULATED	TEMPERATURE OBSERVED	T2 IN DEG F CALCULATED
10.6670	211.65	212.55	184.55	189.40	184.55	189.40
11.1420	212.45	212.60	183.23	193.62	183.23	193.62
11.6170	213.75	212.01	184.00	194.00	184.00	194.00
12.1110	213.44	211.42	184.00	194.03	184.00	194.03
12.6010	214.61	211.97	184.15	193.38	184.15	193.38
13.0150	213.84	211.68	184.57	193.13	184.57	193.13
13.4910	214.79	211.79	185.01	192.74	185.01	192.74
13.9600	215.53	212.11	185.77	192.39	185.77	192.39
14.4380	214.51	211.99	187.53	192.42	187.53	192.42
14.9670	215.57	211.85	190.07	192.72	190.07	192.72
15.3830	212.46	211.74	190.95	192.95	190.95	192.95
15.8370	214.72	211.37	191.39	192.35	191.39	192.35
16.3070	214.60	211.53	191.79	192.12	191.79	192.12
16.7920	215.69	212.09	193.10	192.44	193.10	192.44
17.3150	217.76	211.94	193.60	192.35	193.60	192.35
17.7240	217.93	211.45	194.47	192.61	194.47	192.61
18.2150	219.34	211.61	194.47	193.43	194.47	193.43
18.6580	217.33	211.91	193.57	193.54	193.57	193.54
19.1300	218.65	212.21	193.14	193.24	193.14	193.24

search and the integration of the four differential equations for every function evaluation. Because of this only three runs were analyzed. The results for runs B1 and B2 were much better than those for run B3. On this basis the mean values for θ_{1b} , θ_{2a} and θ_{2b} were taken to be 0.0928, 0.1359 and 0.1763 respectively. Physically, this meant that the model predicted an inside film coefficient which was from 80 to 90 percent higher than that obtained experimentally. This could be either due to the assumptions made in the model or to the empirical nature of the film coefficient correlations (A.16), (A.39) and (A.51).

CHAPTER IV

MINIMUM TIME CONTROL POLICY

This chapter deals with the development of a minimum start-up time control policy for the double effect evaporator using the model equations of Chapter III. The problem is stated in Section IV.1 and this involves identifying state and control variables, equality constraints in the form of algebraic equations, state and control variable inequality constraints, and possible start-up scenarios. Section IV.2 contains the derivation of a general algorithm useful for solving minimum time problems similar to that for the evaporator. The actual use of the algorithm for solving the present problem is described in Section IV.3. It contains the results of model simulations in arriving at the optimal policy for three problems, all of which are minimum time start-up problems under various conditions. Experimental verification of one of the minimum time policies and the effectiveness of the model is presented in Section IV.4.

Refer to the foldout nomenclature list of the more important symbols at the end of this chapter (page 148) to aid in interpreting statements made using these symbols in the succeeding sections.

IV.1 Statement of the Problem for the Evaporator

IV.1.1 State and Control Variables

The state variables, which are necessary to describe completely the state of the process at any particular time, are the differential variables in the model differential equations (3.1) to (3.6) of

Section III.1.1. For uniformity, x_i will be used for a state variable and X will be the state vector with components x_i . These are assigned as follows:

x_1 - First effect hold-up (H_1)

x_2 - First effect liquid enthalpy (h_1)

x_3 - Second effect hold-up (H_2)

x_4 - Second effect liquid enthalpy (h_2)

x_5 - First effect solute concentration (C_1)

x_6 - Second effect solute concentration (C_2).

The decision variables listed in Section III.1.4 have to be defined so that the model is complete. The control or manipulative variables are chosen from this set depending upon the controllability of the process and the physical realizability of the control. For example, the second effect vacuum pressure is not capable of being manipulated physically on this system and so it is not chosen to be a control variable. The feed temperature and concentration are not used as control variables in this problem either. The remaining decision variables, comprising four flow rates and the steam pressure to the first effect, can be easily manipulated physically and can be used to force the process in any desired direction. For example, the feed to the second effect, W_F , and inter-effect flow rate, W'_{12} , can control the inventories in the first and second effects. The steam temperature, T_s , and recirculation rate, W_{11} , have an effect on the first effect temperature and the rate of increase of the second effect temperature. The product flow rate, W_{01} , is used to control the product concentration. These control variables are capable of keeping the process at steady

state and the steady state values for these variables are governed by the steady state solution of the differential equations (3.1) to (3.6).

For uniformity, let u_i denote a control variable and let U be the control vector with components u_i . The assignment is as follows:

u_1 - Feed to the second effect (W_F)

u_2 - Intereffect flow rate (W'_{12})

u_3 - Recirculation flow rate (W_{11})

u_4 - Temperature of steam to first effect (T_S)

u_5 - Product flow rate out of first effect (W_{01}).

Rewriting the state equations (3.1) to (3.6) and the algebraic equations (3.7) to (3.12) in terms of the state and control variable nomenclature defined above, we have

$$\dot{x}_1 = W_{12} - V_{21} - u_3 - u_5 \quad (4.1)$$

$$\dot{x}_2 = \frac{1}{x_1} [W_{12}(h_{12} - x_2) + V_{21}(x_2 - h_1^V) + Q_1] \quad (4.2)$$

$$\dot{x}_3 = u_1 - u_2 - V_{02} \quad (4.3)$$

$$\dot{x}_4 = \frac{1}{x_3} [u_1(h_F - x_4) + V_{02}(x_4 - h_2^V) + Q_2] \quad (4.4)$$

$$\dot{x}_5 = \frac{1}{x_1} [W_{12}(C_{12} - x_5) + V_{21}x_5] \quad (4.5)$$

$$\dot{x}_6 = \frac{1}{x_3} [u_1(C_F - x_6) + V_{02}x_6] \quad (4.6)$$

Connection equations,

$$W_{12} = u_2 + u_3 \quad (4.7)$$

$$W_{12}h_{12} = u_3x_2 + u_2x_4 \quad (4.8)$$

$$U_{12}C_{12} = u_3x_5 + u_2x_6 \quad (4.9)$$

Heat transfer equations,

$$Q_1 = Q_1(x_1, x_2, u_3, u_4, h_{12}, U_{12}, V_{21}) \quad (4.10)$$

$$Q_2 = Q_2(x_2, x_3, x_4, h_F) \quad (4.11)$$

$$V_{21} = Q_2/\lambda_1 \quad (4.12)$$

IV.1.2 State and Control Variable Constraints

It is evident that in a real system the control variables cannot take on all values as there are physical limitations on the maximum and minimum flow rates and temperatures. The lower limit for all the flow variables is zero. The lower limit for the steam temperature is 212°F as steam cannot be supplied at a lower pressure than atmospheric in the first effect. The upper limit depends upon the pipe size and the valve size for the flow rates and on the steam supply pressure for the steam temperature. Thus, all the control variables are subject to lower and upper bounds of the form

$$u_{i,\min} \leq u_i \leq u_{i,\max} \quad (4.13)$$

In a like manner some of the state variables are constrained. At steady state all the state variables should be greater than or equal to their steady state (desired) values (\hat{x}_i)--the steady state hold-ups are the desired operating hold-ups, the steady state temperature in the first effect should be at least the boiling temperature of water at 1 atmosphere, the steady state temperature in the second effect should

be the boiling temperature of water at the pressure in the second effect, and the steady state concentration in the first effect should be equal to the desired concentration. The upper bounds are less clearly defined; for example, the liquid level for the second effect (hold-up) should not exceed the overflow limit. The upper bounds on the temperatures are dictated by the design specifications and by characteristics of the solution being concentrated. In general, the state constraints are given by

$$\hat{x}_i \leq x_i \leq x_{i,\max} \quad (4.14)$$

IV.1.3 Control Scenarios

In the start-up of the evaporator, it is useful to visualize the change with time of the state variables for certain values of the control variables. It is evident that the control variables determine the order in which the state variables reach their steady state or desired values. Intuitively, the optimal policy will endeavor to force each state variable directly to its final steady state value and then maintain it while bringing the others to steady state. The order in which the state variables reach their steady state values completes the scenario. Theoretically, there should be a total of n factorial scenarios for a system with n state variables. But most of these are not possible as certain state variables can reach steady state only if certain others have. For example, in the case of the evaporator, the first effect liquid has to boil before the second effect liquid does. The equations describing a stage in a scenario are different from those describing another stage. For each stage they

are simplifications of the general equations. A typical scenario for the start-up of the evaporator (and the resulting simplifications in the general equations (4.1) to (4.12)) is shown below.

Stage A: $t_0 \leq \text{time } t < t_1$. Filling and heating first effect.

Control variables:

Feed to second effect at maximum flow rate, $u_1 = u_{1,\max}$

All feed delivered directly to first effect, $u_2 = u_1$

Temperature of steam for first effect at maximum value, $u_4 = u_{4,\max}$

No recirculation or product flow possible, $u_3 = u_5 = 0$.

Resulting state equations:

First effect hold-up increasing; $\dot{x}_1 = u_2$, $x_1(t_0) = 0$

Heating of first effect; $\dot{x}_2 = \frac{1}{x_1} [u_2(x_4 - x_2) + Q_1]$, $x_2(t_0) = h_F$

No increase in second effect hold-up; $\dot{x}_3 = 0$, $x_3 = x_3(t_0) = 0$

No heating of second effect; $\dot{x}_4 = 0$, $x_4 = x_4(t_0) = h_F$

No concentration occurring in either effect; $\dot{x}_5 = \dot{x}_6 = 0$;

$$x_5 = x_6 = x_5(t_0) = x_6(t_0) = C_F.$$

Time t_1 , signifying the end of Stage A, is determined when the first effect is filled, i.e., when $x_1(t_1) = \hat{x}_1$.

Stage B: $t_1 \leq \text{time } t < t_2$. First effect solution is being heated and second effect is being filled.

Control variables:

Feed flow to second effect at maximum, $u_1 = u_{1,\max}$

Flow to first effect stopped, $u_2 = 0$

Recirculation flow set at maximum, $u_3 = u_{3,\max}$

Temperature of steam to first effect set at maximum value,

$$u_4 = u_{4,\max}$$

No product withdrawn, $u_5 = 0$.

Resulting state equations:

No change in first effect hold-up; $\dot{x}_1 = 0$, $x_1 = x_1(t_1) = \hat{x}_1$

Heating of first effect; $\dot{x}_2 = Q_1/\hat{x}_1$

Second effect hold-up increasing; $\dot{x}_3 = u_1$, $x_3(t_1) = 0$

No heating of second effect; $\dot{x}_4 = 0$, $x_4 = x_4(t_1) = h_F$

No concentration occurring in either effect; $\dot{x}_5 = \dot{x}_6 = 0$,

$$x_5 = x_6 = x_5(t_1) = x_6(t_1) = C_F.$$

Time t_2 , signifying the end of Stage B, is determined when the second effect is filled, i.e., when $x_3(t_2) = \hat{x}_3$.

Stage C: $t_2 \leq \text{time } t < t_3$. First effect solution is heated to boiling.

Control variables:

Feed flow stopped, $u_1 = 0$

Flow to first effect stopped, $u_2 = 0$

Recirculation flow set at maximum, $u_3 = u_{3,\max}$

Temperature of steam to first effect set at maximum value,

$$u_4 = u_{4,\max}$$

No product withdrawn, $u_5 = 0$.

Resulting state equations:

No change in first effect hold-up; $\dot{x}_1 = 0$, $x_1 = x_1(t_2) = \hat{x}_1$

First effect is being heated; $\dot{x}_2 = Q_1/\hat{x}_1$

No change in second effect hold-up; $\dot{x}_3 = 0$, $x_3 = x_3(t_2) = \hat{x}_3$

No heating of second effect; $\dot{x}_4 = 0$, $x_4 = x_4(t_2) = h_F$

No concentration occurring in either effect; $\dot{x}_5 = \dot{x}_6 = 0$;

$$x_5 = x_6 = x_5(t_2) = x_6(t_2) = C_F.$$

Time t_3 , signifying the end of Stage C, is determined when the first effect starts to boil, i.e., when $x_2(t_3) = \hat{x}_2$.

Stage D: $t_3 \leq \text{time } t < t_4$. First effect solution is boiling (and becoming concentrated). Second effect solution is being heated.

Control variables:

Feed to first effect is equal to vaporization from first effect to maintain constant hold-up, $u_2 = V_{21}$

Feed to second effect set to maintain constant hold-up in second effect, $u_1 = u_2$

Recirculation at maximum, $u_3 = u_{3,\max}$

Temperature of steam to first effect at maximum, $u_4 = u_{4,\max}$

No product withdrawn, $u_5 = 0$.

Resulting state equations:

No change in first and second effect hold-ups; $\dot{x}_1 = \dot{x}_3 = 0$;

$$x_1 = x_1(t_3) = \hat{x}_1; \quad x_2 = x_2(t_3) = \hat{x}_2$$

First effect is being heated; $\dot{x}_2 = \frac{1}{\hat{x}_1} [u_2 x_4 + Q_1 - V_{21} h_1^V]$,

$$x_2(t_3) = \hat{x}_2$$

Second effect is being heated; $\dot{x}_4 = \frac{1}{\hat{x}_3} [u_1 (h_F - x_4) + Q_2]$;

$$x_4(t_3) = h_F$$

First effect solution is being concentrated; $\dot{x}_5 = \frac{1}{\hat{x}_1} (u_2 c_F)$;

$$x_5(t_3) = c_F$$

No concentration of second effect solution; $\dot{x}_6 = 0$; $x_6 = x_6(t_3) = c_F$.

Time t_4 signifying the end of Stage D is determined when the second effect starts to boil, i.e., when $x_4(t_4) = \hat{x}_4$.

Stage E: $t_4 \leq \text{time } t < t_f$. Solution in both effects is boiling and being concentrated.

Control variables:

Feed to first effect set equal to vaporization in first effect to maintain constant hold-up, $u_2 = V_{21}$

Feed to second effect set to maintain constant hold-up in

$$\text{second effect, } u_1 = u_2 + V_{02}$$

Recirculation at maximum, $u_3 = u_{3,\max}$

Temperature of steam to first effect at maximum, $u_4 = u_{4,\max}$

No product withdrawn, $u_5 = 0$.

Resulting state equations:

No change in first or second effect hold-ups; $\dot{x}_1 = \dot{x}_3 = 0$;

$$x_1 = x_1(t_4) = \hat{x}_1, \quad x_3 = x_3(t_4) = \hat{x}_3$$

First effect is being heated; $\dot{x}_2 = \frac{1}{\hat{x}_1} (u_2 x_4 + Q_1 - V_{21} h_1^v)$

No change in second effect enthalpy; $\dot{x}_4 = 0, \quad x_4 = x_4(t_4) = \hat{x}_4$

First effect solution is being concentrated; $\dot{x}_5 = \frac{1}{\hat{x}_1} (u_1 x_6)$

Second effect solution is being concentrated;

$$\dot{x}_6 = \frac{1}{\hat{x}_3} (u_1 C_F - u_2 x_6), \quad x_6(t_4) = C_F.$$

Time t_f , signifying the end of Stage E, and consequently the final time, is determined when the first effect concentration reaches a desired value, i.e., when $x_5(t_f) = \hat{x}_5$.

After the final time, t_f found above, the control variables u_1 , u_2 and u_5 are obtained from the steady state solutions of the differential equations (4.1), (4.3) and (4.5). The feed to the first effect is such that a constant hold-up is maintained in the first effect, i.e., $u_2 = u_5 + V_{21}$. The feed to the second effect is such that a constant hold-up is maintained in the second effect, i.e., $u_1 = u_2 + V_{02}$. The product rate is such that a constant product concentration is obtained, i.e., $u_5 = u_2 \frac{x_6}{x_5}$. Simplifying the above three relationships we can obtain a sequence for determining the control variables u_1 , u_2 and u_5 at steady state as follows:

$$u_2 = v_{21} / \left(1 + \frac{x_6}{x_5} \right) ; \quad u_1 = u_2 + v_{02}; \quad u_5 = u_2 \frac{x_6}{x_5}$$

Note that certain point constraints of the form $x_i(t_j) = \hat{x}_i$ separate the various stages of the scenario. The control variables, state and algebraic equations change when these points in time are encountered.

It is assumed that certain state variables such as the first and second effect hold-ups will be maintained at their steady state values once these are attained. That is, the point constraint $x_i(t_j) = \hat{x}_i$ is followed by the equality constraint $\dot{x}_i = 0$. This latter equality is maintained by calculating the required value of an appropriate control variable. This is similar to the concept of a first order state variable inequality constraint (Bryson et al., 1963), (Bryson and Ho, 1969).

It is clear that another scenario will give rise to a different ordering of the point constraints, $x_i(t_j) = \hat{x}_i$, depending upon the order in which the state variables reach their steady states. The optimal scenario is the one which will result in a minimum final time.

This scenario approach simplifies the mathematical problem to a great extent as a minimum number of state equations have to be integrated. The equations simplify considerably leading to simpler adjoint equations and Hamiltonian minimizations.

IV.1.4 Summary of the Problem Statement

The problem is to minimize the final time, that is

$$\min_U J = t_f$$

subject to the state equations (4.1) to (4.6), the connection equations (4.7) to (4.9), and the heat transfer equations (A.1) to (A.52) where the control variables are constrained,

$$U_{\min} \leq U \leq U_{\max}$$

and point constraints pertaining to the particular scenario are to be satisfied

$$x_i(t_j) = \hat{x}_i ; \quad \begin{array}{l} i = 1, n \\ j = 1, n \end{array}$$

IV.2 A Minimum Time Algorithm

IV.2.1 General Problem

The objective is to minimize the final time by selection of the controls U

$$\underset{U}{\text{Min}} J = t_f \quad (4.15)$$

subject to:

the state equations

$$\dot{X} = f(X, U, Z) ; \quad X(0) \text{ given} \quad (4.16)$$

the algebraic equations

$$g(X, U, Z) = 0 \quad (4.17)$$

the control constraints

$$U_{\min} \leq U \leq U_{\max} \quad (4.18)$$

or

$$\left. \begin{array}{l} h_{1i}(u_i) = u_i - u_{i,\max} \leq 0 \\ h_{2i}(u_i) = u_{i,\min} - u_i \leq 0 \end{array} \right\} h(U) \leq 0 \quad (4.19)$$

and the point constraints

$$\begin{aligned}
 x_0(t_0) &= \hat{x}_0 \\
 x_{k1}(t_{k1}) &= \hat{x}_{k1} \\
 x_{k2}(t_{k2}) &= \hat{x}_{k2} \\
 &\vdots \\
 x_f(t_f) &= \hat{x}_f \\
 t_0 &= \text{fixed}
 \end{aligned} \tag{4.20}$$

IV.2.2 Lagrange Formulation and Necessary Conditions

Assume that the point constraints are met at times t_0, t_{k1}, \dots, t_f . Also assume the times t_i correspond to all those times when one (or more) of the controls or states reaches or leaves a constraint. Next, form the following index sets,

$$\begin{aligned}
 I_0 &= \{0, 1, 2, \dots, k1, k2, \dots, f\} \\
 I_1 &= \{1, 2, \dots, k1, k2, \dots, f\} \\
 I_2 &= \{0, 1, \dots, k1, k2, \dots, f-1\}
 \end{aligned}$$

Within each set, order the indices so the times $t_i, i \in I_0$ or I_1 or I_2 are in increasing order. Also form the index set

$$K = \{k1, k2, \dots\}$$

Introducing multipliers α, β and λ we can write the Lagrangian for the problem

$$\begin{aligned}
 L(X, U, t_i, i \in I_0, \alpha, \beta, \lambda) &= t_f + \alpha_f(\hat{x}_f - x_f(t_f)) + \sum_{k \in K} \alpha_k(\hat{x}_k - x_k(t_k)) \\
 &+ \sum_{i \in I_1} \int_{t_{i-1}}^{t_i} (\lambda^T(f - \dot{x}) + \beta^T h) dt
 \end{aligned} \tag{4.21}$$

The algebraic equations $g(X, U, Z) = 0$ are not included. These will always be satisfied by solving for the dependent variables Z and substituting the resulting values into the state equations. Also the equations at t_0 , $X(t_0) = \hat{X}_0$, will always be satisfied and are not included. Note that the fourth term accounts for the changes in the state equations, algebraic equations, and control variable constraints along the trajectory. This term has been written to allow for possible discontinuities in the Lagrange multipliers, λ , at the entry and exit corners of constraints.

We define the Hamiltonian $H = \lambda^T f$ and rewrite equation (4.21)

$$L(X, U, t_i, i \in I_0, \alpha, \beta, \lambda) = t_f + \alpha_f(\hat{x}_f - x_f(t_f)) \\ + \sum_{k \in K} \alpha_k(\hat{x}_k - x_k(t_k)) + \sum_{i \in I_1} \int_{t_{i-1}^+}^{t_i^-} (H - \lambda^T \dot{X} + \beta^T h) dt \quad (4.22)$$

On an extremal solution, the Lagrangian L must be stationary with respect to small, arbitrary perturbations of the times $t_i (i \in I_1)$; the multipliers α , β , and λ ; and the states X and controls U . To derive the necessary conditions for a stationary point of the Lagrangian, we take first order variations of L with respect to $t_i (i \in I_1)$, α , β , λ , X and U ,

$$\delta L = (1 + H - \lambda^T \dot{X} + \beta^T h - \alpha_f \dot{x}_f)_{t_f} \delta t_f - \alpha_f \delta x_f(t_f) + (\hat{x}_f - x_f(t_f)) \delta \alpha_f \\ + \sum_{k \in K} [-\alpha_k \delta x_k(t_k) + (\hat{x}_k - x_k(t_k)) \delta \alpha_k - (\alpha_k \dot{x}_k)_{t_k} \delta t_k] \\ + \sum_{i \in I_2} [(H - \lambda^T \dot{X} + \beta^T h)_{t_i^-} \delta t_i - (H - \lambda^T \dot{X} + \beta^T h)_{t_{i-1}^+} \delta t_{i-1}]$$

$$\begin{aligned}
& + \sum_{i \in I_1} \int_{t_{i-1}^+}^{t_i^-} \left\{ \left(\frac{\partial H}{\partial \lambda} - \dot{X} \right)^T \delta \lambda + \left(\frac{\partial H}{\partial X^T} + \beta^T \frac{\partial h}{\partial X^T} - \lambda^T \delta \dot{X} \right) \delta X + \left(\frac{\partial H}{\partial U^T} \right. \right. \\
& \left. \left. + \beta^T \frac{\partial h}{\partial U^T} \right) \delta U + h^T \delta \beta \right\} dt
\end{aligned} \tag{4.23}$$

Integrating the term $\int_{t_{i-1}^+}^{t_i^-} -\lambda^T \delta \dot{X} dt$ by parts,

$$\begin{aligned}
- \int_{t_{i-1}^+}^{t_i^-} \lambda^T \delta \dot{X} dt &= (\lambda^T \delta X)_{t_{i-1}^+} - (\lambda^T \delta X)_{t_i^-} + \int_{t_{i-1}^+}^{t_i^-} \dot{\lambda}^T \delta X dt
\end{aligned} \tag{4.24}$$

Substituting (4.24) into (4.23) and collecting coefficients of $\delta \alpha$, $\delta \beta$, $\delta \lambda$, δX , δU , δx_k and δt_i separately, we have

$$\delta L = (1 + H - \lambda^T \dot{X} + \beta^T h - \alpha_f \dot{x}_f)_{t_f} \delta t_f \tag{a}$$

$$- \alpha_f \delta x_f(t_f) - \lambda^T(t_f) \delta X(t_f) \tag{b}$$

$$+ \lambda^T(t_0) \delta X(t_0) \tag{c}$$

$$+ (\hat{x}_f - x_f(t_f)) \delta \alpha_f \tag{d}$$

$$+ \sum_{\substack{i \in I_2 \\ i \neq K}} (\lambda^T)_{t_i^+} \delta X(t_i) - (\lambda^T)_{t_i^-} \delta X(t_i) \tag{e}$$

$$\begin{aligned}
& + \sum_{\substack{i \in I_2 \\ i \neq K}} [(H - \lambda^T \dot{X} + \beta^T h)_{t_i^-} \delta t_i - (H - \lambda^T \dot{X} + \beta^T h)_{t_i^+} \delta t_i] \\
& \tag{f}
\end{aligned}$$

$$+ \sum_{k \in K} [(\lambda^T)_{t_k^+} \delta X(t_k) - (\lambda^T)_{t_k^-} \delta X(t_k) + \alpha_k \delta x_k(t_k)] \tag{g}$$

$$+ \sum_{k \in K} (\hat{x}_k - x_k(t_k)) \delta \alpha_k \quad (h)$$

$$+ \sum_{k \in K} [(H - \lambda^T \dot{x} + \beta^T h)_{t_k^-} \delta t_k - (H - \lambda^T \dot{x} + \beta^T h)_{t_k^+} \delta t_k + (\alpha_k \dot{x}_k)_{t_k} \delta t_k] \quad (o)$$

$$+ \sum_{i \in I_1} \int_{t_{i-1}^+}^{t_i^-} \left[\frac{\partial H}{\partial \lambda} - \dot{x} \right]^T \delta \lambda dt \quad (p)$$

$$+ \sum_{i \in I_1} \int_{t_{i-1}^+}^{t_i^-} \left[\dot{\lambda}^T + \frac{\partial H}{\partial X^T} + \beta^T \frac{\partial h}{\partial X^T} \right] \delta x dt \quad (q)$$

$$+ \sum_{i \in I_1} \int_{t_{i-1}^+}^{t_i^-} \left[\frac{\partial H}{\partial U^T} + \beta^T \frac{\partial h}{\partial U^T} \right] \delta U dt \quad (r)$$

$$+ \sum_{i \in I_1} \int_{t_{i-1}^+}^{t_i^-} h^T \delta \beta dt \quad (s)$$

(4.25)

The Kuhn-Tucker conditions arise from the Kuhn-Tucker multipliers β and the inequality constraints h ,

$$\beta_j h_j = 0 \quad (t)$$

$$\beta \geq 0 \quad (u)$$

For stationarity of the Lagrangian all the coefficients of δx , δU , δx_k , $\delta \alpha$, $\delta \beta$, $\delta \lambda$ and δt_i occurring in equation (4.25) must be zero. By equating these coefficients to zero we arrive at the following necessary conditions (NC).

$$\text{From term (q); } \dot{\lambda}^T = -\frac{\partial H}{\partial \lambda^T} - \beta^T \frac{\partial h}{\partial \lambda^T}; t_{i-1}^+ \leq t \leq t_i^- \quad (\text{NC1})$$

$$\text{From term (r); } \frac{\partial H}{\partial U^T} + \beta^T \frac{\partial h}{\partial U^T} = 0; t_{i-1}^+ \leq t \leq t_i^- \quad (\text{NC2})$$

$$\text{From term (p); } \dot{X} = \frac{\partial H}{\partial \lambda}; f(X, U); t_{i-1}^+ \leq t \leq t_i^- \quad (\text{NC3})$$

$$\text{From term (t); } \beta_j h_j = 0; t_{i-1}^+ \leq t \leq t_i^- \quad (\text{NC4})$$

$$\text{From term (u); } \beta \geq 0; t_{i-1}^+ \leq t \leq t_i^- \quad (\text{NC5})$$

$$\text{From term (h); } \hat{x}_k - x_k(t_k) = 0; k \in K \quad (\text{NC6})$$

$$\text{From term (d); } \hat{x}_f - x_f(t_f) = 0 \quad (\text{NC7})$$

$$\begin{aligned} \text{From term (b); } \lambda_f(t_f) &= -\alpha_f(t_f) \\ \lambda_j(t_f) &= 0; j=1, \dots, n; j \neq f \end{aligned} \quad (\text{NC8})$$

$$\text{From term (c); } \lambda_j(t_0) = \text{unknown since } \delta x(t_0) = 0 \quad (\text{NC9})$$

$$\begin{aligned} \text{From term (e); } \lambda_j(t_i^+) &= \lambda_j(t_i^-); j=1, \dots, n \\ i \in I_2, i \notin K \end{aligned} \quad (\text{NC10})$$

$$\begin{aligned} \text{From term (g); } \lambda_j(t_k^+) &= \lambda_j(t_k^-); j=1, \dots, n; j \neq k; k \in K \\ \lambda_k(t_k^+) &= \lambda_k(t_k^-) - \alpha_k; k \in K \end{aligned} \quad (\text{NC11})$$

$$\text{From term (a), (NC4), (NC5) and (NC8); } H(t_f) = -1 \quad (\text{NC12})$$

$$\text{From term (f), (NC4) and (NC11); } H(t_i^-) = H(t_i^+); i \in I_2, i \notin K \quad (\text{NC13})$$

$$\text{From term (o), (NC4) and (NC12); } H(t_k^-) = H(t_k^+); k \in K \quad (\text{NC14})$$

IV.2.3 Comments on the Necessary Conditions

The state equations and point constraints are the necessary conditions NC3, NC6 and NC7. The Kuhn-Tucker conditions NC4 and NC5 indicate that, when a control constraint h is encountered, that constraint is held by choosing U such that $h(X, U) = 0$ provided that the Kuhn-Tucker multiplier β is non-negative.

The problem is set up such that the point constraints arise when there is a change in the form of the state equations and, consequently, a change in the adjoint equations and Hamiltonian. For example, before time t_1 (where $x_1(t_1) = \hat{x}_1$) let $f^{(1)}$ represent the state equations, or

$$\dot{X} = f^{(1)}(X, U) \quad t < t_1$$

After time t_1 let $f^{(2)}$ represent the state equations, that is

$$\dot{X} = f^{(2)}(X, U) \quad t > t_1$$

The Hamiltonians are

$$H^{(1)} = \lambda^T f^{(1)} \quad \text{and} \quad H^{(2)} = \lambda^T f^{(2)}$$

The functional forms $f^{(1)}$ and $f^{(2)}$ reflect the change in the state equations. Condition NC14 shows that the Hamiltonian is continuous across the times t_k where point constraints are encountered. For example, at $t = t_1$, $x_1(t_1) = \hat{x}_1$ and $H^{(1)} = H^{(2)}$.

Condition NC11 states that the Lagrange multiplier (or adjoint variable), corresponding to the particular state on which there is a point constraint at t_i , has a discontinuity at t_i , whereas the

multipliers corresponding to the other states are continuous at t_i .

At time t_1 , for example,

$$\lambda_1(t_1^+) = \lambda_1(t_1^-) - \alpha_1$$

$$\lambda_j(t_1^+) = \lambda_j(t_1^-) ; \quad j=2,\dots,n$$

The value of the "jump," α_1 , in the multiplier λ_1 , is readily determined from the condition $H^{(1)} = H^{(2)}$.

At times t_i , $i \in I_0$, when control constraints have to be satisfied, all the multipliers are continuous as the trajectories enter and leave the constraints--NC10. The Hamiltonian is also continuous at such points in time as shown by condition NC13.

Necessary conditions NC8 and NC12 provide values for the adjoint variables and Hamiltonian at the final time. The adjoint variables corresponding to the states that are unconstrained at the final time have a value zero. If only one state is constrained at the final time, the adjoint variable corresponding to this state can be determined from the final condition on the Hamiltonian

$$H(t_f) = -1$$

or

$$\lambda_f \dot{x}_f = -1$$

However, if more than one state variable is constrained at the final time, the values for all but one adjoint variable have to be guessed and the remaining one adjoint variable can then be determined from the final condition on the Hamiltonian. Let us suppose that the adjoint variable corresponding to the stopping condition $x_f(t_f) = \hat{x}_f$ is determined from the final value of the Hamiltonian and that another

variable λ_i has been guessed. On subsequent iterations this value of λ_i is updated by noting that the gradient of L with respect to $\lambda_i(t_f)$ is $-(\hat{x}_i - x_i(t_f))$. The Saddle Point Theorem requires us to maximize L with respect to $\lambda_i(t_f)$ which is equivalent to driving the gradient to zero. So the value of $\lambda_i(t_f)$ on following iterations should be in such a direction that at the final time the deviation of $x_i(t_f)$ from the desired value x_i , i.e., $(\hat{x}_i - x_i(t_f))$ is driven to zero.

This same iterative technique is used if more than one point constraint is encountered at an intermediate point in time t_k . One of the adjoint variables at time t_k^- can be determined by using the continuity of the Hamiltonian at time t_k . The other variables will have to be guessed initially and then updated using the gradients available on subsequent iterations.

Necessary condition NC2, along with the Kuhn-Tucker conditions NC4 and NC5 enables us to replace condition NC2 by another condition made possible by invoking the strong minimum principle. Denn (1969) has lucidly shown how the Hamiltonian takes on its minimum value for the optimal decision function $U(t)$, both on and off the inequality constraints. Utilizing this result, we can replace the stationary condition NC2 by

$$\min_U H(X, U, \lambda)$$

subject to $U_{\min} \leq U \leq U_{\max}$.

IV.2.4 Minimum Time Algorithm

An algorithm for arriving at the minimum time policy was

developed based on the necessary conditions arrived at in Section IV.2.2. It can be classified as a "Minimum H" algorithm as it deals with direct Hamiltonian minimizations along the trajectory. Proposing a control scenario to start with is essential to simplify the problem and to use the algorithm effectively.

Let the various elements of the state vector be divided into 3 groups as follows:

Group A: States which remain unconstrained through start-up.

Group B: States which meet their steady state values during start-up and which define the point constraints.

Group C: States which meet their steady state values at the final time.

The state equations are modified along the trajectory when the point constraint conditions are met by the variables in Group B. Also, all algebraic equations are satisfied throughout the trajectory, and this is implied in the state equations. When a variable belonging to Group B arrives at its steady state value, it is assumed that for subsequent time the state equation is replaced by the corresponding algebraic equation $\dot{x} = 0$. Thus the Hamiltonian $H = \lambda^T f$ is different for points on the trajectory depending upon which of the state equations are active; so also are the adjoint equations

$$\dot{\lambda} = -\frac{\partial H}{\partial X^T} - \beta^T \frac{\partial h}{\partial X^T} ; \quad t_{i-1}^+ \leq t \leq t_i^-$$

The algorithm proceeds as follows:

1) Guess a nominal control policy U which will cause a stopping condition, $x_f = \hat{x}_f$, to be satisfied at the final time.

2) Integrate the state equations forward until the stopping condition is satisfied. This implicitly determines the final time. The proper set of state equations should be integrated depending upon the point constraints, the control constraints, and the algebraic equations which have to be satisfied along the trajectory.

3) At the final time determined in step (2), let all the multipliers of the variables in Group A be zero. Guess the multipliers of all the variables in Group C with the exception of that corresponding to the stopping condition. Determine this latter multiplier from the final condition on the Hamiltonian,

$$\lambda_f \dot{x}_f = -1 - \sum_{\substack{i \in C \\ i \neq f}} \lambda_i \dot{x}_i$$

4) With the values of the adjoint variables at the final time determined in step (3), integrate the adjoint equations in the reverse direction. At times t_k , when point constraints were met on the forward integration, determine the values $\lambda(t_k^-)$ by utilizing the continuity and jump conditions on the adjoint variables and Hamiltonian. For example, if only one point constraint of the form $x_k(t_k) = \hat{x}_k$ is met at time t_k , then $\lambda_k(t_k^-)$ can be determined from the continuity of the Hamiltonian

$$\lambda_k(t_k^-) f_k(t_k^-) + \sum_{\substack{j \in n \\ j \neq k}} \lambda_j(t_k^-) f_j(t_k^-) = \sum_{\substack{j \in n \\ j \neq k}} \lambda_j(t_k^+) f_j(t_k^+)$$

and $\lambda_j(t_k^-) = \lambda_j(t_k^+)$; $j \in n$, $j \neq k$. If more than one point constraint is met at time t_k , then the values of all but one of these multipliers should be guessed at time t_k^- and the last one determined from the continuity of the Hamiltonian.

5) Simultaneously on the reverse iteration, minimize the Hamiltonian $H = \lambda^T f$ at each point to determine the optimal U^*

$$\min_{U^*} H(X, U, \lambda)$$

6) On reaching time t_0 , update the control policy used on the forward integration with that found on the reverse integration in step (5)

$$U^{i+1} = U^i + \zeta(t)(U^* - U^i)$$

$\zeta(t)$ is chosen to limit the change in U if too large a change is indicated.

7) Integrate the state equations forward as in step (2). At the final time determine the difference in the states for the variables in Group C from their desired values and update the guess on the multipliers λ_i such that the gradient of L with respect to $\lambda_i(t_f)$ is driven to zero, i.e., to drive $(\hat{x}_i - x_i(t_f))$ to zero. As before, determine the λ corresponding to the stopping condition from the final value of the Hamiltonian.

8) Integrate the adjoint equations in the reverse direction as in step (4). Update the guess on $\lambda_j(t_k^-)$; $j \in n$, $j \neq k$ in a similar manner as in step (7). Determine λ_k as before from the continuity of the Hamiltonian.

9) Repeat steps (5)-(8) until

(1) $\delta J < \varepsilon_1$ - no significant improvement in the final time

(2) $\|U^{i+1} - U^i\| < \varepsilon_2$ - no significant change in the control policy

and (3) $\|\delta \lambda_j\| < \eta$ at t_f

and (4) $\|\delta \lambda_j\| < \eta$ at t_k^- .

The optimal policy is chosen as the one which satisfies the above conditions.

IV.3 Solution to the Evaporator Problem

IV.3.1 Problem 1. Constraint on the Second Effect Hold-up

The problem was solved for the scenario described in Section IV.1.3. The concentration equations (4.5) and (4.6) were not used in this simulation. The Hamiltonians and adjoint equations for the various stages of the scenario are given below.

The general Hamiltonian for the problem is:

$$H = \lambda_1 (W_{12} - V_{21} - u_3) + \frac{\lambda_2}{x_1} [W_{12}(h_{12} - x_2) + V_{21}(x_2 - h_1^v) + Q_1] \\ + \lambda_3 (u_1 - u_2 - V_{02}) + \frac{\lambda_4}{x_3} [u_1(h_F - x_4) + V_{02}(x_4 - h_2^v) + Q_2]$$

This simplifies for the various stages as follows:

Stage A: $t_0 < t < t_1$

$$H_a = \lambda_1 u_2 + \frac{\lambda_2}{x_1} [u_2(x_4 - x_2) + Q_1] + \lambda_3 (u_1 - u_2)$$

Stage B: $t_1 < t < t_2$. $x_1(t_1) = \hat{x}_1$

$$H_b = \frac{\lambda_2}{\hat{x}_1} Q_1 + \lambda_3 (u_1 - u_2)$$

$$W_{12} = u_3 ; u_2 = 0 \text{ or } f_1 = 0$$

Stage C: $t_2 < t < t_3$ $x_3(t_2) = \hat{x}_3$

$$H_c = \frac{\lambda_2}{\hat{x}_1} Q_1$$

$$u_1 = u_2 \text{ or } f_3 = 0 \text{ and } f_1 = 0$$

$$\text{Stage D: } t_3 < t < t_f \quad x_2(t_3) \geq \hat{x}_2$$

$$H_d = \frac{\lambda_2}{\hat{x}_1} [W_{12}(h_{12} - x_2) + V_{21}(x_2 - h_1^V) + Q_1] \\ + \frac{\lambda_4}{\hat{x}_3} [u_1(h_F - x_4) + Q_2]$$

$$W_{12} = V_{21} + u_2 \quad \text{or} \quad f_1 = 0; \quad u_1 = u_2 \quad \text{or} \quad f_3 = 0$$

Time t_f is determined when the second effect starts to boil, i.e., when $x_4(t_f) = \hat{x}_4$.

The variables Q_1 , Q_2 , V_{21} , V_{02} , W_{12} , h_{12} , h_1^V , h_F are determined from the algebraic equations in Appendix A.

The adjoint equations are

$$\dot{\lambda}_i = - \frac{\partial H}{\partial x_i}, \quad i = 1, 2, 3, 4$$

and H is either H_a , H_b , H_c or H_d .

The partial derivatives $\frac{\partial H}{\partial x}$ are evaluated numerically for each point in time using the appropriate Hamiltonian valid at that time. Note that in Stage A, $\lambda_4 = 0$; this is because the liquid enthalpy in the second stage, x_4 , is constrained to stay at $x_4 = h_F$ by $\dot{x}_4 = 0$. In Stage B, $\lambda_1 = 0$ since the first effect hold-up $x_1 = \hat{x}_1 = \text{constant}$. That is, the objective function is made insensitive to changes in x_1 by holding $x_1 = \hat{x}_1$. Likewise, in Stage C, $\lambda_3 = 0$ and $\lambda_1 = 0$ because both holdups are fixed.

The final time is determined when the second effect boils, i.e., when $x_4(t_f) = \hat{x}_4$. At t_f , λ_2 and λ_4 are active. $\lambda_2 = 0$ since the final condition on the liquid enthalpy in Stage 1, x_2 , is unspecified. λ_4 is

determined from the final condition on the Hamiltonian. On the reverse integration of the adjoint variables at time t_2 the second effect holdup, x_3 , becomes constrained. The multiplier $\lambda_3(t_2^-)$ is found from continuity of the Hamiltonian $H_b(t_2^-) = H_c(t_2^+)$. At time t_1 , the first effect holdup, x_1 , becomes constrained and $\lambda_1(t_1^-)$ is determined from the continuity of the Hamiltonian at time t_1 , $H_a(t_1^-) = H_b(t_1^+)$. Also, on the reverse integration, the appropriate Hamiltonian for each stage is minimized with respect to the control variables U . Note that in Stage A minimization of H_a is with respect to second effect feed u_1 , intereffect flow u_2 , and first effect steam temperature u_4 . The recirculation rate u_3 and the product flow u_5 cannot be started when the first effect is still filling. In Stage 3 the feed flow to the first effect, u_2 , has a set value as it maintains the first effect holdup $x_1 = \hat{x}_1$. So minimization of H_b is with respect to u_2 , u_3 and u_4 . Likewise, in Stage C, the second effect feed flow, u_1 , is fixed to maintain the second effect holdup at $x_3 = \hat{x}_3$ and minimization of H_c is with respect to the recirculation flow u_3 and the steam temperature u_4 . The same holds for Stage D.

Minimization of the Hamiltonian was achieved by a slightly modified version of the computer program VA04A originally coded by M. J. D. Powell. It is based on a conjugate gradient search technique which does not involve partial derivatives, and it is explained in Fletcher and Powell (1963). This routine was adapted for bounded variable minimizations by writing a package which accounted for the control variable constraints. When a control constraint was encountered, a perturbation of the control variable into the feasible region decides

whether the constraint should be held or be released. This is basically a numerical evaluation of the associated Kuhn-Tucker multiplier.

The forward and reverse integrations were done using Hamming's predictor-corrector method HPCG available in the IBM Scientific Subroutine Package. The adjoint equations required the numerical values of partial derivatives, and a subroutine PDERIV was written to do this.

Tables 4.1 to 4.6 and Figures 4.1 to 4.3 are the results of the three iterations to determine the optimal policy. The control policy, shown at the top of each figure, is the one used during the forward iteration resulting in the states shown. The adjoint variables on the reverse iteration are also shown at the bottom of each figure. The final time decreased from 13.2 minutes to 10.27 minutes in three iterations. The maximum allowed change in the flow rates were 1 lb/min for u_1 and u_2 and 10 lbs/min for the recirculation rate u_3 . The maximum allowed change in the steam temperature was 5°F. The minimum time for this scenario could not be reduced any further as all the variables were bounded.

It can be noticed from the plots that the final control policy is bang-bang and that the switches occur at the point constraints assumed in the scenario. This is indeed a fortunate result as the control policy can be put in feedback form dependent upon whether the states are below or at their steady state values. This is not a typical result but is due to the assumed control scenario which is specific to this type of problem. The final control policy which resulted in a minimum time can thus be put into the following feedback form.

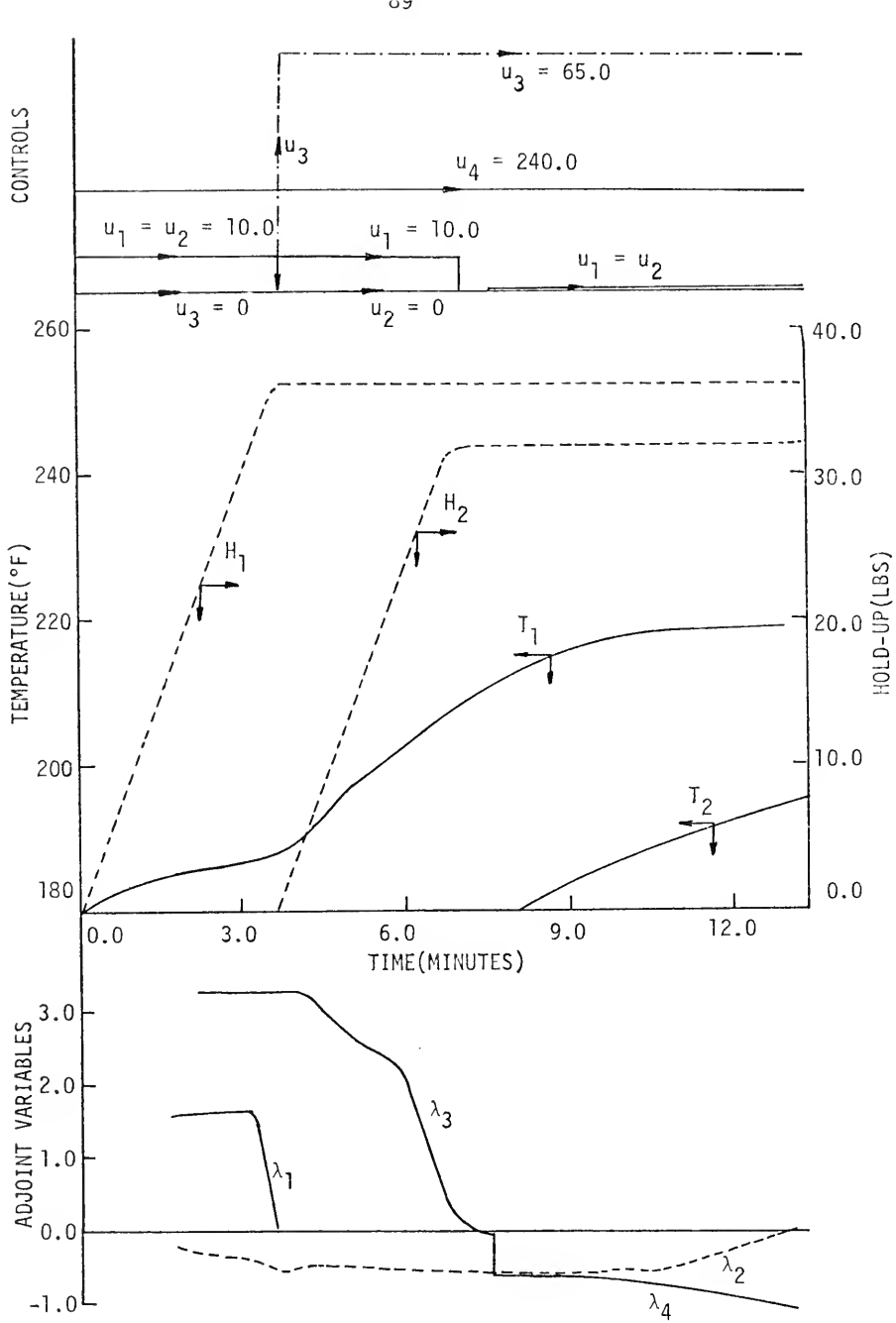


Figure 4.1 Control, State and Adjoint Variables for Problem 1, Iteration 1

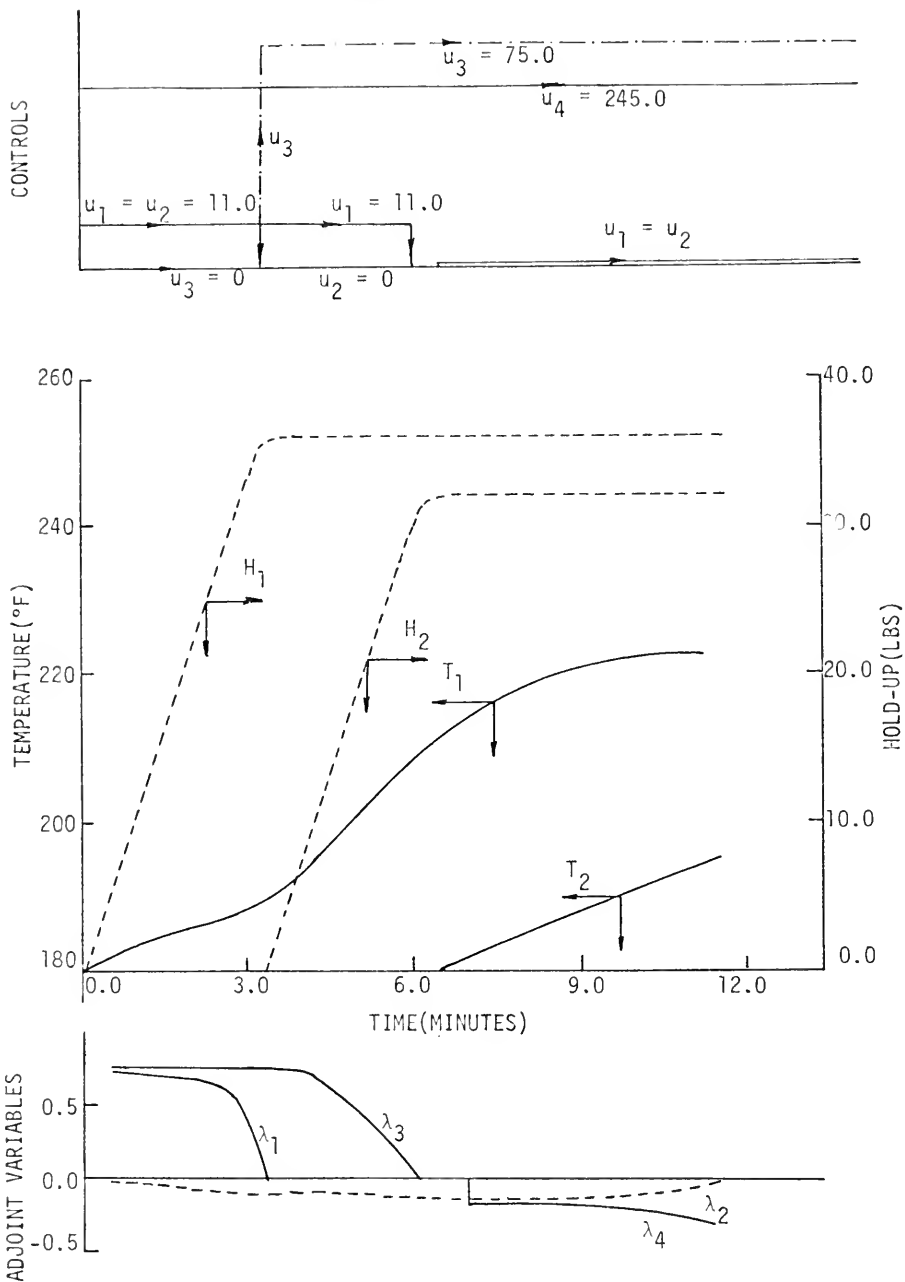


Figure 4.2 Control, State and Adjoint Variables for Problem 1, Iteration 2

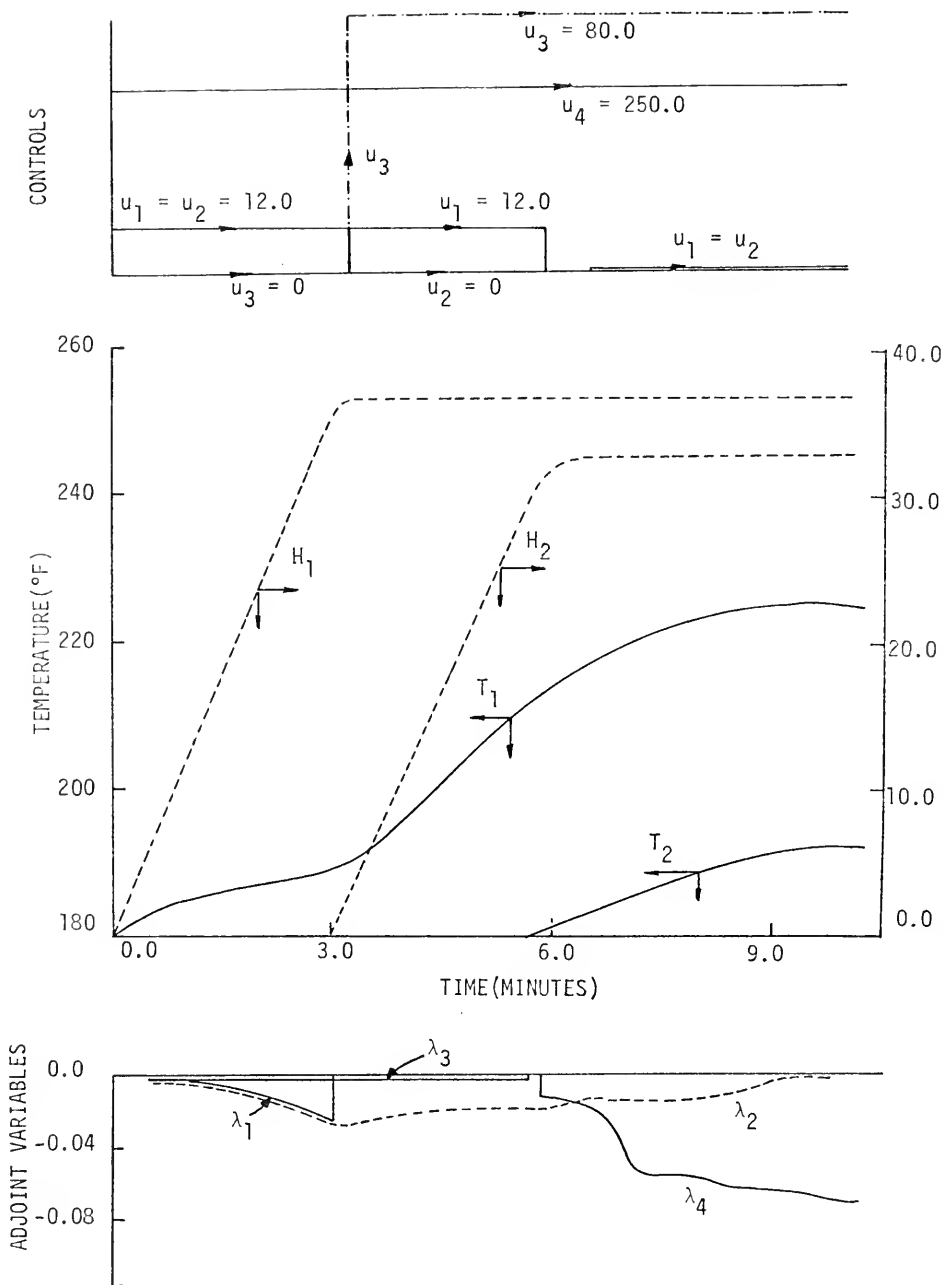


Figure 4.3 Control, State and Adjoint Variables for Problem 1, Iteration 3

TABLE 4.1

STATE VARIABLES FOR PROBLEM 1, ITERATION 1

TIME (MINS)	S T A T E X1 (LBS)	V A R I A B L E S X2 (BTU/LB)	X3 (LBS)	X4 (BTU/LB)	C O N T R O L U1 (L B S / M I N)	U2	V A R I A B L E S U3 (D E G F)	U4 (D E G F)
0.0	0.0	147.92	0.0	147.92	10.00	10.00	0.0	240.00
0.50	5.00	149.59	0.07	147.92	10.00	10.00	0.0	240.00
1.00	10.00	151.80	0.06	147.92	10.00	10.00	0.0	240.00
1.50	15.00	153.39	0.07	147.92	10.00	10.00	0.0	240.00
2.00	20.00	153.91	0.07	147.92	10.00	10.00	0.0	240.00
2.50	25.00	154.34	0.07	147.92	10.00	10.00	0.0	240.00
3.00	30.00	154.67	0.06	147.92	10.00	10.00	0.0	240.00
3.50	35.00	154.84	0.07	147.92	10.00	10.00	0.0	240.00
4.34	36.16	160.32	7.34	147.96	10.00	0.0	65.00	240.00
4.66	36.16	162.51	10.46	147.94	10.00	0.0	65.00	240.00
5.03	36.16	165.01	14.21	147.94	10.00	0.0	65.00	240.00
5.41	36.16	167.39	17.96	147.93	10.00	0.0	65.00	240.00
5.91	36.16	170.36	22.96	147.93	10.00	0.0	65.00	240.00
6.66	36.16	174.43	30.46	147.93	10.00	0.0	65.00	240.00
6.91	36.16	175.70	32.10	147.93	0.0	0.0	65.00	240.00
7.28	36.16	177.51	31.98	147.93	0.0	0.0	65.00	240.00
7.78	36.16	179.69	31.97	148.85	0.09	0.09	65.00	240.00
8.03	36.16	180.59	31.97	149.56	0.09	0.09	65.00	240.00
8.41	36.16	181.74	31.97	150.65	0.09	0.09	65.00	240.00
8.66	36.16	182.41	31.97	151.38	0.09	0.09	65.00	240.00
8.91	36.16	183.01	31.97	152.09	0.09	0.09	65.00	240.00
9.16	36.16	183.56	31.97	152.80	0.09	0.09	65.00	240.00
9.66	36.16	184.50	31.97	154.21	0.09	0.09	65.00	240.00
10.66	36.16	186.00	31.97	156.92	0.09	0.09	65.00	240.00
11.16	36.16	186.61	31.97	158.22	0.09	0.09	65.00	240.00
12.16	36.16	187.63	31.97	160.71	0.08	0.08	65.00	240.00
13.16	36.16	186.69	31.97	163.81	0.33	0.25	65.00	240.00

TABLE 4.2

ADJOINT VARIABLES FOR PROBLEM 1, ITERATION 1

TIME (MINS)	A D J O LAMBDA1	I N T LAMBDA2	V A R I A B L E S LAMBDA3	LAMBDA4	C O N T R O L U1	(L B S / M I N)	V A R I A B L E S U3	(DEG F)	U4
13.16	0.0	0.0	0.0	-1.1016	0.32	0.32	80.00	240.80	
12.02	0.0	-0.2434	0.0	-0.9567	0.31	0.31	80.00	240.10	
11.27	0.0	-0.4184	0.0	-0.8513	0.32	0.32	80.00	240.30	
10.77	0.0	-0.5294	0.0	-0.7826	0.32	0.32	80.00	240.70	
10.39	0.0	-0.5936	0.0	-0.7412	0.33	0.33	80.00	240.50	
9.77	0.0	-0.5415	0.0	-0.7007	0.33	0.33	80.00	240.40	
8.77	0.0	-0.6011	0.0	-0.6382	0.35	0.35	80.00	241.90	
8.14	0.0	-0.5582	0.0	-0.6041	0.31	0.31	80.00	232.90	
7.47	0.0	-0.5669	-0.0040	0.0					
6.64	0.0	-0.5642	0.4749	0.0	0.0	0.26	80.00	250.00	
5.64	0.0	-0.5385	2.3590	0.0	0.0	0.09	80.00	250.00	
5.02	0.0	-0.5055	2.6100	0.0	0.0	0.07	80.00	250.00	
4.23	0.0	-0.4904	3.2973	0.0	0.0	0.0	80.00	250.00	
3.64	-0.0006	-0.5208	3.2982	0.0					
3.19	1.6895	-0.4526	3.2982	0.0	0.0	12.00	0.0	250.00	
2.69	1.6734	-0.3771	3.2982	0.0	0.0	12.00	0.0	250.00	
2.19	1.6513	-0.3026	3.2982	0.0	0.0	12.00	0.0	250.00	
1.89	1.6352	-0.2581	3.2982	0.0					

TABLE 4.3
STATE VARIABLES FOR PROBLEM 1, ITERATION 2

TIME (MINS)	S T A T E x1 (LBS)	x2 (BTU/LB)	x3 (LBS)	x4 (BTU/LB)	C O N T R O L U1	U2 (L B S / M I N)	V A R I A B L E S U3	U4 (DEG F)
0.0	0.0	147.92	0.0	147.92	11.00	11.00	0.0	245.00
0.50	5.50	149.96	0.07	147.92	11.00	11.00	0.0	245.00
1.00	11.00	152.33	0.06	147.92	11.00	11.00	0.0	245.00
1.50	16.50	153.78	0.07	147.92	11.00	11.00	0.0	245.00
2.00	22.00	154.31	0.07	147.92	11.00	11.00	0.0	245.00
2.50	27.50	154.80	0.07	147.92	11.00	11.00	0.0	245.00
3.00	33.00	155.10	0.06	147.92	11.00	11.00	0.0	245.00
3.13	34.37	155.16	0.06	147.92	11.00	11.00	0.0	245.00
3.59	35.93	158.23	3.67	147.92	11.00	0.0	75.00	245.00
4.00	36.16	161.58	7.91	147.92	11.00	0.0	75.00	245.00
4.63	36.16	166.64	14.78	147.92	11.00	0.0	75.00	245.00
5.00	36.16	169.46	18.91	147.92	11.00	0.0	75.00	245.00
5.50	36.16	172.97	24.41	147.92	11.00	0.0	75.00	245.00
6.00	36.16	176.21	29.91	147.92	11.00	0.0	75.00	245.00
6.25	36.16	177.73	31.70	147.92	0.0	0.0	75.00	245.00
6.69	36.16	180.68	32.12	148.95	0.10	0.10	75.00	245.00
7.06	36.16	182.69	32.35	150.06	0.10	0.10	75.00	245.00
7.31	36.16	183.80	32.82	150.78	0.10	0.10	75.00	245.00
7.56	36.16	184.76	32.81	151.54	0.10	0.10	75.00	245.00
8.06	36.16	186.37	33.71	152.93	0.11	0.11	75.00	245.00
8.56	36.16	187.64	33.79	154.41	0.10	0.10	75.00	245.00
9.06	36.16	188.68	33.72	155.89	0.10	0.10	75.00	245.00
9.31	36.16	189.13	33.72	156.62	0.10	0.10	75.00	245.00
9.81	36.16	189.93	33.72	158.05	0.10	0.10	75.00	245.00
10.31	36.15	190.62	33.72	159.45	0.10	0.10	75.00	245.00
10.81	36.15	191.23	33.73	160.80	0.09	0.09	75.00	245.00
11.31	36.15	190.53	33.72	162.42	0.56	0.37	75.00	245.00

TABLE 4.4

ADJOINT VARIABLES FOR PROBLEM 1, ITERATION 2

TIME (MINS)	A D J O I N T LAMBDA1	LAMBDA2	V A R I A B L E S LAMBDA3	LAMBDA4	C O N T R O L U1	U2 (L B S / M I N)	V A R I A B L E S U3 (D E G F)	U4
11.31	0.0	0.0	0.0	-0.2800	0.04	0.04	80.00	250.00
10.88	0.0	-0.0825	0.0	-0.2307	0.35	0.35	80.00	250.00
10.32	0.0	-0.1195	0.0	-0.2081	0.35	0.35	80.00	246.90
9.76	0.0	-0.1014	0.0	-0.1906	0.36	0.36	80.00	245.08
9.20	0.0	-0.1246	0.0	-0.1762	0.36	0.36	80.00	246.90
8.32	0.0	-0.1426	0.0	-0.1642	0.37	0.37	80.00	246.90
7.57	0.0	-0.1071	0.0	-0.1530	0.38	0.38	80.00	245.26
7.07	0.0	-0.1126	0.0	-0.1403	0.31	0.31	80.00	250.00
6.63	0.0	-0.1138	0.0	0.0	0.29	0.29	80.00	250.00
6.20	0.0	-0.1137	-0.0101	0.0	0.0	0.25	80.00	250.00
5.66	0.0	-0.1121	0.2647	0.0	0.0	0.0	80.00	250.00
4.37	0.0	-0.0960	0.6850	0.0	0.0	0.0	0.0	0.0
3.95	0.0	-0.0958	0.7514	0.0	0.0	0.0	0.0	0.0
3.39	-0.0017	-0.1033	0.7508	0.0	0.0	12.00	0.0	250.00
2.50	0.6438	-0.0794	0.7508	0.0	0.0	12.00	0.0	250.00
1.50	0.6853	-0.0461	0.7508	0.0	0.0	12.00	0.0	250.00
0.50	0.7307	-0.0145	0.7508	0.0	0.0	12.00	0.0	250.00

TABLE 4.5
STATE VARIABLES FOR PROBLEM 1, ITERATION 3

TIME (MINS)	S T A T E V A R I A B L E S				C O N T R O L V A R I A B L E S			
	X1 (LBS)	X2 (BTU/LB)	X3 (LBS)	X4 (BTU/LB)	U1 (LBS / M I N)	U2	U3	U4 (DEG F)
0.0	0.0	147.92	0.0	147.92	12.00	12.00	0.0	250.00
0.50	6.00	150.34	0.07	147.92	12.00	12.00	0.0	250.00
1.00	12.00	152.91	0.06	147.92	12.00	12.00	0.0	250.00
1.50	18.00	154.17	0.07	147.92	12.00	12.00	0.0	250.00
2.00	24.00	154.72	0.07	147.92	12.00	12.00	0.0	250.00
2.50	30.00	155.26	0.07	147.92	12.00	12.00	0.0	250.00
2.75	33.00	155.42	0.07	147.92	12.00	12.00	0.0	250.00
2.88	34.50	155.49	0.07	147.92	12.00	12.00	0.0	250.00
3.09	35.82	156.68	1.37	147.92	12.00	0.0	80.00	250.00
3.50	36.18	160.48	5.88	147.92	12.00	0.0	80.00	250.00
3.88	36.18	164.08	10.39	147.92	12.00	0.0	80.00	250.00
4.13	36.18	166.37	13.39	147.92	12.00	0.0	80.00	250.00
4.38	36.18	168.57	16.39	147.92	12.00	0.0	80.00	250.00
4.86	36.18	172.71	22.39	147.92	12.00	0.0	80.00	250.00
5.13	36.18	174.66	25.39	147.92	12.00	0.0	80.00	250.00
5.38	36.18	176.53	28.39	147.92	12.00	0.0	80.00	250.00
5.84	36.18	181.12	32.87	148.89	0.10	0.10	80.00	250.00
6.25	36.18	184.27	32.85	150.13	0.11	0.11	80.00	250.00
6.50	36.18	185.81	32.85	150.93	0.11	0.11	80.00	250.00
6.75	36.18	187.12	32.85	151.76	0.11	0.11	80.00	250.00
7.25	36.18	189.22	32.85	153.43	0.12	0.12	80.00	250.00
7.75	36.18	190.84	32.85	155.12	0.12	0.12	80.00	250.00
8.25	36.18	192.13	32.85	156.80	0.11	0.11	80.00	250.00
9.25	36.18	194.07	32.85	160.05	0.11	0.11	80.00	250.00
9.88	36.18	192.79	32.85	159.93	0.10	0.10	80.00	250.00
10.25	36.18	193.64	32.85	161.09	0.10	0.10	80.00	250.00
10.38	36.18	193.90	32.85	161.47	1.10	0.47	80.00	250.00

TABLE 4.6

ADJOINT VARIABLES FOR PROBLEM 1, ITERATION 3

TIME (MINS)	A D J O I N T LAMBDA1	V A R I A B LAMBDA2	V A R I A B LAMBDA3	L A M B D A 4	C O N T R O L U1	U2 (L B S / M I N)	V A R I A B U3	L E S U4 (D E G F)
10.17	0.0	0.0	0.0	-0.0692	0.16	0.16	87.00	250.00
9.17	0.0	-0.0067	0.0100	-0.0646	0.17	0.17	87.00	250.00
8.67	0.0	-0.0108	0.0	-0.0614	0.17	0.17	87.00	250.00
8.17	0.0	-0.0127	0.0	-0.0590	0.18	0.18	87.00	250.00
7.92	0.0	-0.0135	0.0	-0.0578	0.19	0.19	87.00	250.00
7.42	0.0	-0.0149	0.0	-0.0557	0.18	0.18	87.00	250.00
7.17	0.0	-0.0154	0.0	-0.0546	0.19	0.19	87.00	250.00
6.92	0.0	-0.0158	0.0	-0.0537	0.19	0.19	87.00	250.00
6.67	0.0	-0.0160	0.0	-0.0240	0.0	0.0	87.00	250.00
6.36	0.0	-0.0154	0.0	-0.0173	0.0	0.0	87.00	250.00
6.17	0.0	-0.0167	0.0	-0.0175	0.0	0.0	87.00	250.00
5.70	0.0	-0.0172	-0.0050	-0.0156	12.00	0.0	87.00	250.00
5.27	0.0	-0.0183	-0.0010	-0.0130	12.00	0.0	87.00	250.00
4.27	0.0	-0.0214	-0.0010	-0.0071	12.00	0.0	87.00	250.00
3.64	0.0	-0.0235	-0.0010	-0.0032	12.00	0.0	87.00	250.00
3.10	-0.0221	-0.0243	-0.0010	-0.0019	12.00	12.00	7.0	250.00
2.75	-0.0200	-0.0214	-0.0010	-0.0019	12.00	12.00	7.0	250.00
2.00	-0.0148	-0.0153	-0.0010	-0.0019	12.00	12.00	7.0	250.00
1.50	-0.0101	-0.0112	-0.0010	-0.0019	12.00	12.00	7.0	250.00
1.00	-0.0038	-0.0073	-0.0010	-0.0019	12.00	12.00	7.0	250.00

Intereffect liquid flow, $u_2 = u_{2,\max}$ for $x_1 < \hat{x}_1$ (first effect filling)

Intereffect liquid flow, $u_2 = v_{21}$ for $x_1 \geq \hat{x}_1$ (first effect filled)

Feed flow, $u_1 = u_{1,\max}$ for $x_3 < \hat{x}_3$ (second effect filling)

Feed flow, $u_1 = u_2 + v_{02}$ for $x_3 \geq \hat{x}_3$ (second effect filled)

Recirculation rate, $u_3 = u_{3,\max}$ for $x_1 \geq \hat{x}_1$ (recirculation after first effect filled)

Steam temperature, $u_4 = u_{4,\max}$ for all time (input steam temperature at maximum)

With the optimal policy as shown above a simulation of the state equations including the concentration equations was done and is shown in Figure 4.4 while Table 4.7 gives the simulated values. The product flow rate, u_5 , was kept shut off until the desired concentration was reached in the first effect. While concentration was taking place, the two hold-ups were kept steady by maintaining

$$u_2 = v_{21} \quad \text{and} \quad u_1 = u_2 + v_{02}$$

After the desired concentration \hat{C}_1 was achieved, the product flow rate was fixed at $u_5 = u_2 \hat{C}_2 / C_1$ to keep the product concentration constant.

In the simulation shown in Figure 4.4, it was assumed that the initial and final concentrations of the solute were very small so as to cause a negligible boiling point elevation and also so that the properties of water were not altered by the presence of the solute. For example, a 3 percent weight solution of sodium hydroxide would cause a boiling point elevation of approximately 3°F at 212°F which was considered to be an insignificant elevation for this problem.

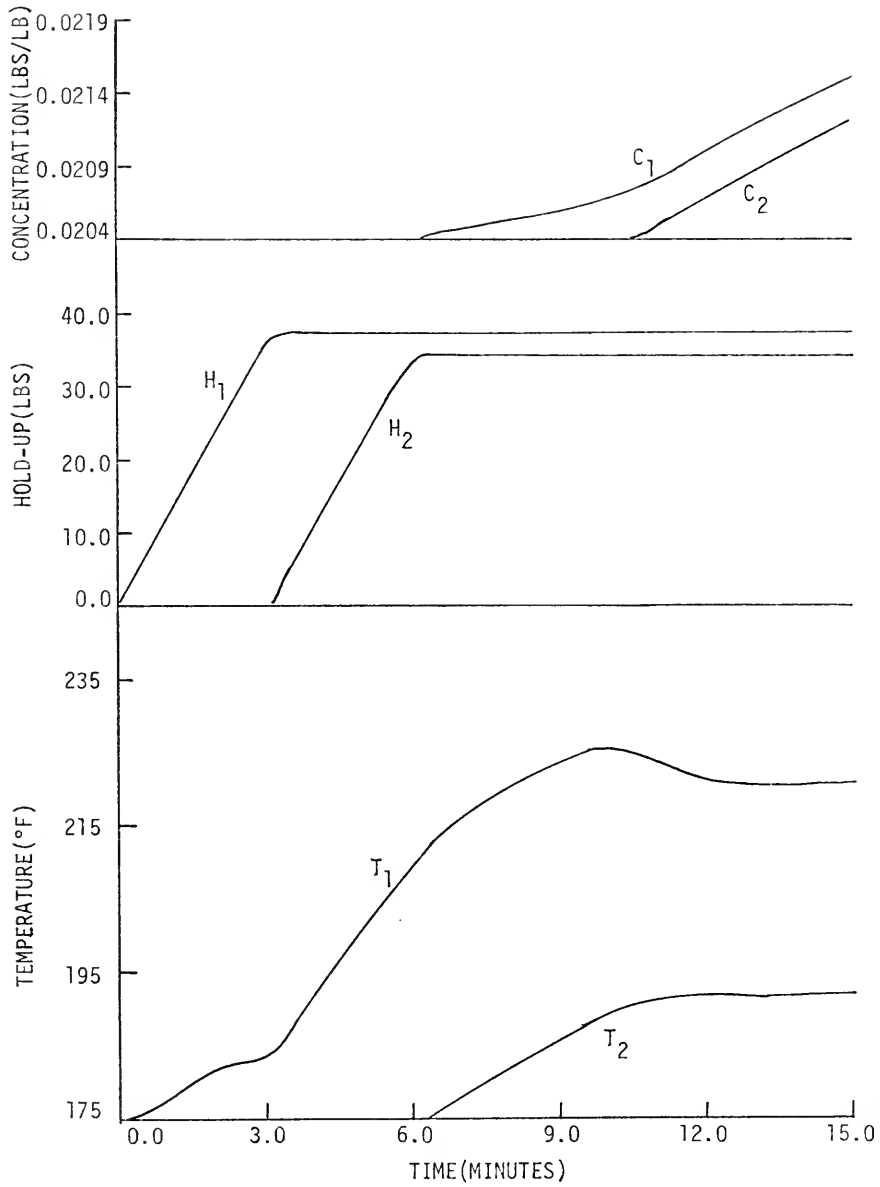


Figure 4.4 Optimal Simulation including Concentration dynamics

TABLE 4.7 - CONTINUED

TIME	S T A T E		V A R I A B L E S		C O N T R O L		V A R I A B L E S				
	X1	X2	X3	X4	X5	X6	U1	U2	U3	U4	U5
6.13	37.19	178.09	34.08	143.41	0.02040	0.02040	0.0	0.0	80.00	250.00	0.0
6.31	37.19	179.86	34.06	143.89	0.02041	0.02040	0.11	0.11	80.00	250.00	0.0
6.50	37.19	181.50	34.05	144.52	0.02042	0.02040	0.12	0.12	80.00	250.00	0.0
6.69	37.19	182.93	34.06	145.15	0.02043	0.02040	0.12	0.12	80.00	250.00	0.0
6.81	37.19	183.77	34.06	145.59	0.02044	0.02040	0.12	0.12	80.00	250.00	0.0
6.94	37.19	184.55	34.06	146.03	0.02045	0.02040	0.13	0.13	80.00	250.00	0.0
7.06	37.19	185.27	34.06	146.48	0.02046	0.02040	0.13	0.13	80.00	250.00	0.0
7.31	37.19	186.55	34.06	147.38	0.02048	0.02040	0.13	0.13	80.00	250.00	0.0
7.56	37.19	187.64	34.06	148.29	0.02049	0.02040	0.13	0.13	80.00	250.00	0.0
7.81	37.19	188.60	34.06	149.21	0.02051	0.02040	0.13	0.13	80.00	250.00	0.0
8.06	37.18	189.44	34.06	150.12	0.02053	0.02040	0.13	0.13	80.00	250.00	0.0
9.06	37.18	191.99	34.06	153.74	0.02060	0.02040	0.13	0.13	80.00	250.00	0.0
10.06	37.18	193.74	34.06	157.18	0.02067	0.02040	0.12	0.12	80.00	250.00	0.0
11.31	37.18	189.51	34.06	159.27	0.02087	0.02057	0.09	0.09	80.00	250.00	0.0
11.81	37.18	189.36	34.06	159.50	0.02095	0.02065	0.79	0.39	80.00	250.00	0.0
12.31	37.18	189.40	34.06	159.51	0.02104	0.02073	0.79	0.39	80.00	250.00	0.0
12.81	37.18	188.84	34.06	159.47	0.02113	0.02083	0.76	0.38	80.00	250.00	0.0
13.31	37.18	188.96	34.06	159.86	0.02121	0.02089	0.76	0.38	80.00	250.00	0.0
13.81	37.18	188.11	34.06	159.59	0.02131	0.02100	0.73	0.36	80.00	250.00	0.0
14.31	37.18	188.07	34.06	159.48	0.02140	0.02109	0.73	0.36	80.00	250.00	0.0
14.81	37.18	188.03	34.06	159.44	0.02149	0.02118	0.73	0.36	80.00	250.00	0.0
15.06	37.18	188.26	34.06	159.54	0.02153	0.02121	0.73	0.37	80.00	250.00	0.36

IV.3.2 Problem 2. Fixed Feed Rate

The minimum start-up time was again determined using the same algorithm for a problem similar to Problem 1 but with the added restriction that the feed rate was fixed at a nominal value. Such a start-up procedure is used by the students in the undergraduate unit operations laboratory. It was desired to determine the effect of this policy. (No verification was made that industrial practice uses this procedure, but it would be surprising if many start-up policies did not.) The nominal value chosen was 2.9 lbs/min (0.025 kg/s) which is the steady state flow rate used for the laboratory experiment. It is clear that the minimum time must be at least 23 minutes as this is the time needed just to fill the two effects. The product flow rate from the first effect then became an active control variable and had to be used to maintain a constant hold-up in the first effect. The control scenario was modified for this problem as the final time was determined by the hold-up in the second effect reaching its steady state value. The second effect solution started boiling at an earlier time and so was not used as the stopping condition as in Problem 1. The scenario was as follows:

Stage A: $t_0 \leq \text{time } t < t_1$. First and second effects are being filled. First effect is being heated.

Control variables:

Recirculation flow, u_3 , and product flow, u_5 , are not possible since the first effect hold-up has not reached its desired value.

Feed to first effect, u_2 , and steam temperature, u_4 , are found from minimization of the Hamiltonian.

Hamiltonian:

$$H_a = \lambda_1 u_2 + \frac{\lambda_2}{x_1} [u_2(x_4 - x_2) + Q_1] + \lambda_3(W_F - u_2)$$

The second effect enthalpy is constant; $\dot{x}_4 = 0$ or $x_4 = h_F$ resulting in $\lambda_4 = 0$. Time t_1 , signifying the end of Stage A, is determined when the first effect is filled, i.e., when $x_1(t_1) = \hat{x}_1$.

Stage B: $t_1 \leq \text{time } t < t_2$. First effect is filled and is being heated. Second effect is being filled.

Control variables:

Feed to the first effect, $u_2 = 0$ to maintain a constant hold-up. Product from first effect, $u_5 = 0$. Minimization of the Hamiltonian determines the recirculation rate, u_3 , and the steam temperature, u_4 .

Hamiltonian:

$$H_b = \frac{\lambda_2}{\hat{x}_1} Q_1 + \lambda_3 W_F$$

The first effect hold-up and the second effect enthalpy are unchanged; $\dot{x}_1 = \dot{x}_4 = 0$; and $\lambda_1 = \lambda_4 = 0$. Time t_2 , signifying the end of Stage B, is determined when the first effect liquid starts to boil, i.e., when $x_2(t_2) = \hat{x}_2$.

Stage C: $t_2 \leq \text{time } t < t_3$. Second effect is being filled and being heated.

Control variables:

The feed to the first effect, u_2 , is such that it maintains a constant first effect hold-up, $u_2 = V_{21}$. The product rate $u_5 = 0$. The recirculation rate, u_3 , and the steam temperature, u_4 , are determined from minimization of the Hamiltonian.

Hamiltonian:

$$H_c = \frac{\lambda_2}{\hat{x}_1} [W_{12}(h_{12} - x_2) + V_{21}(x_2 - h_1^V) + Q_1] + \lambda_3(W_F - u_2) \\ + \frac{\lambda_4}{x_3} [W_F(h_F - x_4) + Q_2]$$

Time t_3 , signifying the end of Stage C, is determined when the second effect starts to boil, i.e., when $x_4(t_3) = \hat{x}_4$.

Stage D: $t_3 \leq \text{time } t < t_f$. Second effect is being filled.

Control variables:

Same as in Stage C.

Hamiltonian:

$$H_d = \frac{\lambda_2}{\hat{x}_1} [W_{12}(h_{12} - x_2) + V_{21}(x_2 - h_1^V) + Q_1] + \lambda_3(W_F - u_2 - V_{02})$$

First effect hold-up and second effect enthalpy are maintained constant.

Time t_f , is determined when the second effect hold-up reaches its desired value, i.e., when $x_3(t_f) = \hat{x}_3$.

The final condition on the Hamiltonian yields a value for

$\lambda_3(t_f)$

$$\lambda_2(t_f) = 0, \lambda_3(t_f)\dot{x}_3(t_f) = -1 \text{ determines } \lambda_3(t_f)$$

At intermediate points continuity of the Hamiltonian is used to determine the unknown multipliers. At time t_3 ,

$$H_d(t_3^+) = H_c(t_3^-) \text{ determines } \lambda_4(t_3^-)$$

At time t_1 ,

$$H_b(t_1^+) = H_a(t_1^-) \text{ determines } \lambda_1(t_1^-)$$

It took 5 iterations for the final time to reach its minimum

value of 24.53 minutes. The feed rate was fixed at 2.9 lbs/minute (0.025 kg/s). The control variable which contributed significantly to this reduction in the final time was the intereffect flow rate u_2 . The state variables for three of the iterations are shown in Figure 4.5 and the values are tabulated in Tables 4.8 to 4.10. The results of the iterations were.

<u>Iteration #</u>	<u>u_2</u>	<u>Final Time</u>
1	1.7	26.875
2	2.0	26.43
3	2.2	25.35
4	2.4	24.625
5	2.9	24.531

Again, it can be noticed from the plots that the switches in the control variables take place at point constraints, i.e., the switches can be directly related to the states yielding a feedback control policy. The control policy can be put in feedback form as follows:

Intereffect liquid rate, $u_2 = W_F$ for $x_1 < \hat{x}_1$ (first effect filling)

Intereffect liquid rate, $u_2 = V_{21}$ for $x_1 \geq \hat{x}_1$ and $x_3 < \hat{x}_3$ (first effect filled, second effect filling)

Intereffect liquid rate, $u_2 = W_F - V_{02}$ for $x_3 \geq \hat{x}_3$ (second effect filled)

Recirculation rate, $u_3 = 0$ for $x_1 < \hat{x}_1$ (first effect filling)

Recirculation rate, $u_3 = u_{3,\max}$ for $x_1 \geq \hat{x}_1$ (first effect filled)

Steam temperature, $u_4 = u_{4,\max}$ for $x_2 < \hat{x}_2$ (first effect heating)

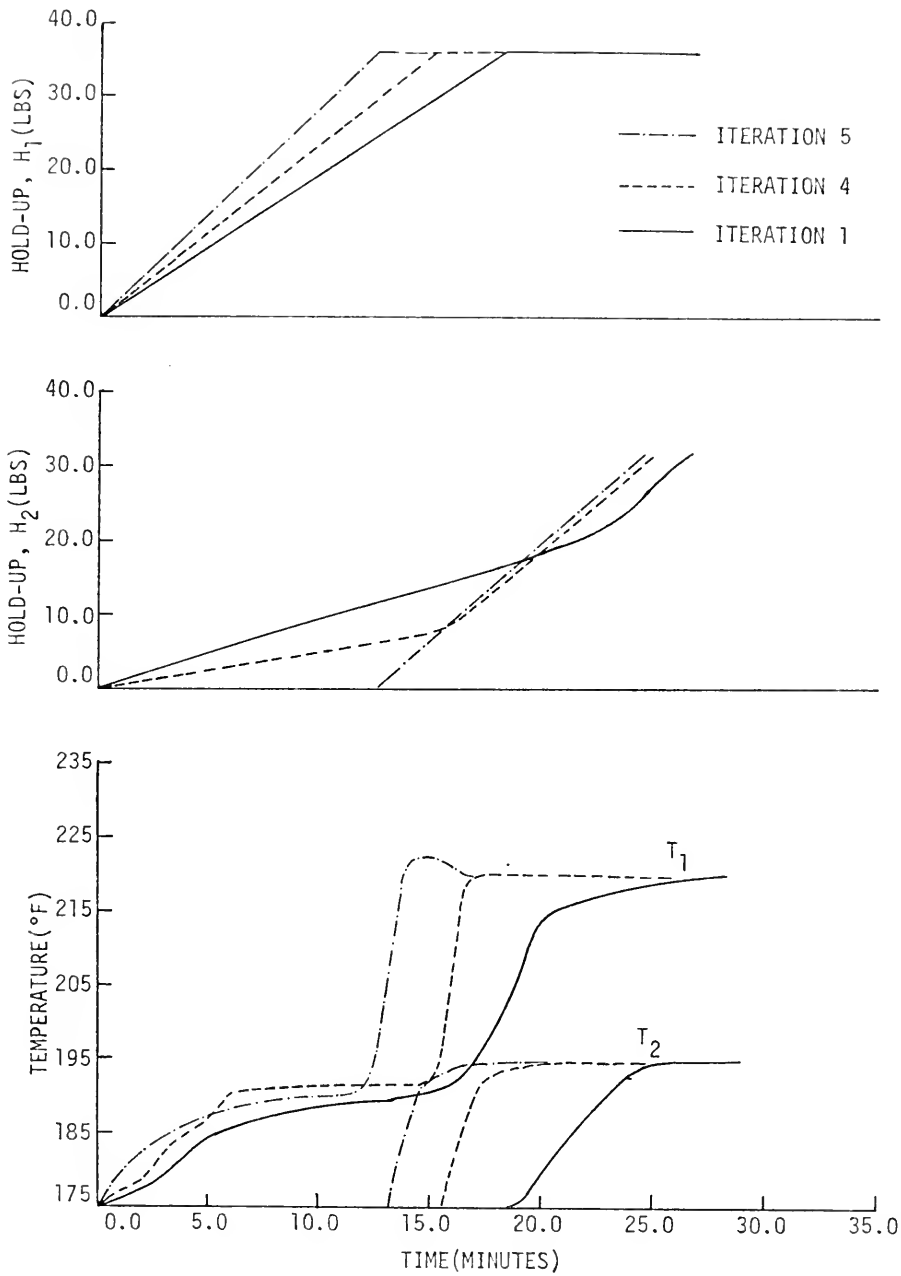


Figure 4.5 State Variables for Problem 2, Iterations 1, 4 and 5

TABLE 4.8

STATE VARIABLES FOR PROBLEM 2, ITERATION 1

TIME (MINS)	S T A T E		V A R I A B L E S		C O N T R O L			V A R I A B L E S	
	X1 (LBS)	X2 (BTU/LB)	X3 (LBS)	X4 (BTU/LB)	U1	U2 (L B S / M I N)	U3	U4 (DEG F)	
0.0	0.0	143.50	0.0	143.50	0.0	2.00	0.0	245.00	
1.00	2.00	144.46	0.89	143.50	0.0	2.00	0.0	245.00	
2.13	4.25	145.76	2.01	143.50	0.0	2.00	0.0	245.00	
3.13	6.25	148.16	3.01	143.50	0.0	2.00	0.0	245.00	
4.13	8.25	150.62	4.01	143.50	0.0	2.00	0.0	245.00	
5.13	10.25	152.75	5.01	143.50	0.0	2.00	0.0	245.00	
6.63	13.25	154.64	6.51	143.00	0.0	2.00	0.0	245.00	
8.63	17.25	156.16	8.51	143.50	0.0	2.00	0.0	245.00	
10.63	21.25	157.07	10.51	143.50	0.0	2.00	0.0	245.00	
12.63	25.25	157.67	12.51	143.50	0.0	2.00	0.0	245.00	
14.63	29.25	158.10	14.51	143.50	0.0	2.00	0.0	245.00	
15.63	31.25	159.61	15.51	143.50	0.0	2.00	0.0	245.00	
16.63	33.25	161.32	16.51	143.50	0.0	2.00	0.0	245.00	
17.63	35.25	165.82	17.51	143.50	0.0	2.00	0.0	245.00	
18.84	36.12	173.15	18.21	143.50	0.0	0.0	75.00	245.00	
19.52	36.14	181.85	18.49	145.54	0.0	0.02	75.00	245.00	
20.80	36.34	183.78	19.61	150.39	0.0	0.04	75.00	245.00	
21.92	36.34	184.85	20.93	154.54	0.0	0.13	75.00	245.00	
22.92	36.24	185.83	22.80	158.26	0.0	0.17	75.00	245.00	
23.89	36.34	186.00	24.72	161.11	0.0	0.24	75.00	245.00	
24.89	36.30	186.32	27.23	161.33	0.0	0.26	75.00	245.00	
26.02	36.24	187.41	30.57	161.64	0.0	0.26	75.00	245.00	
26.88	36.34	187.54	31.69	162.34	0.0	0.26	75.00	245.00	

TABLE 4.9

STATE VARIABLES FOR PROBLEM 2, ITERATION 4

TIME (MINS)	S T A T E	V A R I A B L E S	C O N T R O L	V A R I A B L E S	U4 (DEG F)
	X1 (LBS)	X2 (BTU/LB)	X3 (LBS)	X4 (BTU/LB)	
0.0	0.0	143.50	0.0	143.41	0.0
0.50	1.20	142.93	0.23	143.41	0.0
1.00	2.40	145.10	0.48	143.41	0.0
2.00	4.80	146.44	0.98	143.41	0.0
3.00	7.20	150.45	1.48	143.41	0.0
4.00	9.60	152.95	1.98	143.41	0.0
5.00	12.00	154.53	2.48	143.41	0.0
6.00	14.40	158.35	2.98	143.41	0.0
7.00	16.80	158.59	3.48	143.41	0.0
8.00	19.20	159.01	3.98	143.41	0.0
9.00	21.60	158.98	4.48	143.41	0.0
10.00	24.00	159.02	4.98	143.41	0.0
11.00	26.40	159.14	5.48	143.41	0.0
12.00	28.80	159.26	5.98	143.41	0.0
13.00	31.20	159.38	6.48	143.41	0.0
14.00	33.60	159.46	6.98	143.41	0.0
15.00	36.00	159.54	7.48	143.41	0.0
16.13	37.15	180.46	9.57	149.58	80.00
16.94	37.15	187.95	11.78	160.23	0.0
18.03	37.15	187.91	14.62	161.89	0.0
19.13	37.15	187.86	17.46	162.34	0.0
20.00	37.15	188.70	19.73	162.75	0.0
21.09	37.15	187.85	22.57	162.65	0.0
22.19	37.14	187.97	25.41	162.54	0.0
23.06	37.14	187.92	27.67	162.48	0.0
23.94	37.14	188.14	29.94	162.55	0.0
24.63	37.23	187.22	31.34	162.29	2.51

TABLE 4.10

STATE VARIABLES FOR PROBLEM 2, ITERATION 5

TIME (MINS)	S T A T E V A R I A B L E S				C O N T R O L			V A R I A B L E S			
	X1 (LBS)	X2 (BTU/LB)	X3 (LBS)	X4 (BTU/LB)	U1	U2 (L B S / M I N)	U3	U4 (DEG F)			
0.0	0.0	143.50	0.0	143.50	0.0	2.90	0.0	250.00			
1.00	2.90	143.80	0.0	143.50	0.0	2.90	0.0	250.00			
2.22	6.43	151.80	0.0	143.50	0.0	2.90	0.0	250.00			
3.13	9.06	153.51	0.0	143.50	0.0	2.90	0.0	250.00			
4.27	12.37	155.16	0.0	143.50	0.0	2.90	0.0	250.00			
5.41	15.68	156.08	0.0	143.50	0.0	2.90	0.0	250.00			
6.78	19.66	156.76	0.0	143.50	0.0	2.90	0.0	250.00			
7.91	22.93	157.13	0.0	143.50	0.0	2.90	0.0	250.00			
9.08	25.32	157.41	0.0	143.50	0.0	2.90	0.0	250.00			
10.88	31.53	157.72	0.0	143.50	0.0	2.90	0.0	250.00			
12.25	35.35	157.91	0.0	143.50	0.0	2.90	0.0	250.00			
12.91	36.74	166.75	0.0	143.50	0.0	0.0	80.00	250.00			
13.28	36.79	180.38	1.78	143.55	0.0	0.01	80.00	220.00			
13.66	36.79	189.96	2.87	148.44	0.0	0.07	80.00	220.00			
14.03	36.78	190.89	3.95	156.25	0.0	0.13	80.00	220.00			
14.47	36.79	189.05	5.18	157.93	0.0	0.19	80.00	220.00			
15.03	36.79	187.62	6.73	159.73	0.0	0.25	80.00	220.00			
15.78	36.79	188.29	8.71	160.05	0.0	0.36	80.00	220.00			
17.06	36.78	188.35	12.04	161.23	0.0	0.36	80.00	220.00			
18.81	36.68	187.97	16.57	161.45	0.0	0.36	80.00	220.00			
19.91	36.78	188.11	19.42	162.24	0.0	0.36	80.00	220.00			
20.78	36.78	187.98	21.69	162.35	0.0	0.36	80.00	220.00			
21.88	36.78	187.93	24.53	162.48	0.0	0.36	80.00	220.00			
22.97	36.78	188.25	27.37	162.82	0.0	0.36	80.00	220.00			
24.06	36.78	188.21	30.20	162.89	0.0	0.36	80.00	220.00			
24.53	36.78	188.14	31.32	162.91	0.0	2.14	80.00	220.00			

Steam temperature, $u_4 = u_{4,\min}$ for $x_2 \geq \hat{x}_2$ (first effect boiling)

Product rate, $u_5 = 0$ for $x_3 < \hat{x}_3$ (second effect filling)

Product rate, $u_5 = u_2 - V_{21}$ for $x_3 \geq \hat{x}_3$ (second effect filled)

IV.3.3 Problem 3. No Bound on the Second Effect Hold-up

The minimum time problem was run again for the conditions as in Problem 1 but with the upper bound on the second effect hold-up removed. The second effect hold-up H_2 (state variable x_3) was permitted to vary with the restriction that it should be within one percent of its desired steady state value at the final time. Since the hold-up is allowed to vary the feed to the second effect, u_1 , need not be equal to the flow of liquid and vapor out of the second effect, $u_2 + V_{02}$, and minimization has to be done with respect to u_1 and u_2 as opposed to u_2 alone as in problem 1. The control scenario is different from the one used in problem 1 as we now assume that $x_4(t_f) = \hat{x}_4$ and $x_3(t_f) = \hat{x}_3 \pm 0.01 \hat{x}_3$ at the final time. In other words two of the states reach steady state at the final time resulting in an iterative solution to update the guess on one of the adjoint variables at the final time. The scenario is as follows:

Stage A: $t_0 \leq \text{time } t < t_1$. The two effects are being filled while the first effect is being heated.

Control variables:

The first effect recirculation flow, u_3 , and product flow, u_5 , are not possible until the first effect hold-up has reached its steady state value. Thus $u_3 = u_5 = 0$.
Minimization of the Hamiltonian is with respect to the

feed to the second effect, u_1 , the intereffect flow rate, u_2 , and the steam temperature, u_4 .

Hamiltonian:

$$H_a = \lambda_1 u_2 + \frac{\lambda_2}{x_1} [u_2(x_4 - x_2) + Q_1] + \lambda_3(u_1 - u_2)$$

The second effect enthalpy is unchanged, $\dot{x}_4 = 0$, $x_4 = h_F$, $\lambda_4 = 0$.

Time t_1 , signifying the end of Stage A, is determined when the first effect hold-up reaches its steady state value, i.e., $x_1(t_1) = \hat{x}_1$.

Stage B: $t_1 \leq \text{time } t < t_2$. First effect is being heated and the second effect is being filled.

Control variables:

The feed to the first effect is stopped, i.e., $u_2 = 0$, to maintain the first effect hold-up at \hat{x}_1 . The product rate u_5 is kept off, i.e., $u_5 = 0$. Minimization of the Hamiltonian is with respect to the second effect feed rate, u_1 , the first effect recirculation rate, u_3 , and the steam temperature, u_4 .

Hamiltonian:

$$H_b = \frac{\lambda_2}{\hat{x}_1} [u_2(x_4 - x_2) + Q_1] + \lambda_3(u_1 - u_2)$$

The first effect hold-up, x_1 , is unchanged; $\dot{x}_1 = 0$, $x_1 = \hat{x}_1$,

$\lambda_1 = 0$. The second effect enthalpy is also unchanged;

$\dot{x}_4 = 0$, $x_4 = h_F$, $\lambda_4 = 0$.

Time t_2 , signifying the end of Stage B is found from the condition $x_2(t_2) = \hat{x}_2$, that is when the enthalpy of the solution in the first effect reaches its steady state value.

Stage C: $t_2 \leq \text{time } t < t_f$. The second effect is being filled and heated.

Control variables:

The feed to the first effect, u_2 , is set to maintain the first effect hold-up constant, $u_2 = V_{21}$. The product rate, u_5 , is kept at zero. Minimization of the Hamiltonian is with respect to the feed to the second effect, u_1 , the first effect recirculation rate, u_3 , and the steam temperature, u_4 .

Hamiltonian:

$$H_c = \frac{\lambda_2}{\hat{x}_1} [W_{12}(h_{12} - x_2) + V_{21}(x_2 - h_1^V) + Q_1] + \lambda_3(u_1 - u_2) + \frac{\lambda_4}{x_3} [u_1(h_F - x_4) + Q_2]$$

The first effect hold-up is unchanged; $\dot{x}_1 = 0$, $x_1 = \hat{x}_1$,
 $\lambda_1 = 0$.

Time t_f , the final time is determined when the second effect solution starts to boil, i.e., $x_4(t_f) = \hat{x}_4$. At this time we also require that the second effect hold-up reach a certain value $x_3(t_f) = \hat{x}_3 \pm 0.01\hat{x}_3$.

This change in the scenario requires that one of the adjoint variables at the final time, $\lambda_3(t_f)$, be guessed initially and then updated on subsequent iterations. The multiplier $\lambda_4(t_f)$, corresponding to the stopping condition $x_4(t_f) = \hat{x}_4$ (solution starts to boil), is found from the final value of the Hamiltonian. $\lambda_3(t_f)$ is updated on successive iterations by choosing it to zero the gradient of L with respect to it, which is equivalent to driving $\hat{x}_3 - x_3(t_f)$ to zero.

Tables 4.11 to 4.15 and Figures 4.6 to 4.8 show the results of three of the iterations. Of interest is that t_f is decreased from 10.27 minutes to 9.80 minutes. The initial policy was taken to be the final policy from Problem 1. On the reverse iteration $\lambda_3(t_f)$ (corresponding to the second effect hold-up) was guessed to be zero. This resulted in a control policy that was bang-bang on all the variables but which required the switching time (T_{sw}) on the second effect feed rate, u_1 , to be increased. The results for iterations 2 through 7 are summarized below.

Iteration	T_{sw}	u_1		t_f	$\hat{x}_3 - x_3(t_f)$	$\lambda_3(t_f)$
		$t < T_{sw}$	$t > T_{sw}$			
2	5.875	$u_{1,max}$	$u_{1,min}$	10.375	-2.9	0.17
3	5.85	$u_{1,max}$	$u_{1,min}$	10.25	-2.0	0.09
4	5.8	$u_{1,max}$	$u_{1,min}$	10.13	-0.34	0.04
5	5.75	$u_{1,max}$	$u_{1,min}$	10.13	-0.07	0.01
6	5.7	$u_{1,max}$	$u_{1,min}$	9.8	0.0	0.005
7	5.6	$u_{1,max}$	$u_{1,min}$	9.75	2.92	

The second effect feed flow rate, u_1 , was switched from its maximum value to its minimum value at time T_{sw} . Integration of the state equations was continued until the second effect started to boil, i.e., when $x_4(t_f) = \hat{x}_4$. At this time t_f , the deviation of $x_3(t_f)$ from its desired value \hat{x}_3 was computed and a correction applied to $\lambda_3(t_f)$ as shown in the right-most column. Convergence was obtained at the switching time 5.7 minutes as this had a minimum final time as well as a minimum deviation of the second effect hold-up, x_3 , from its steady value, \hat{x}_3 . Any further reduction in the switching time did result

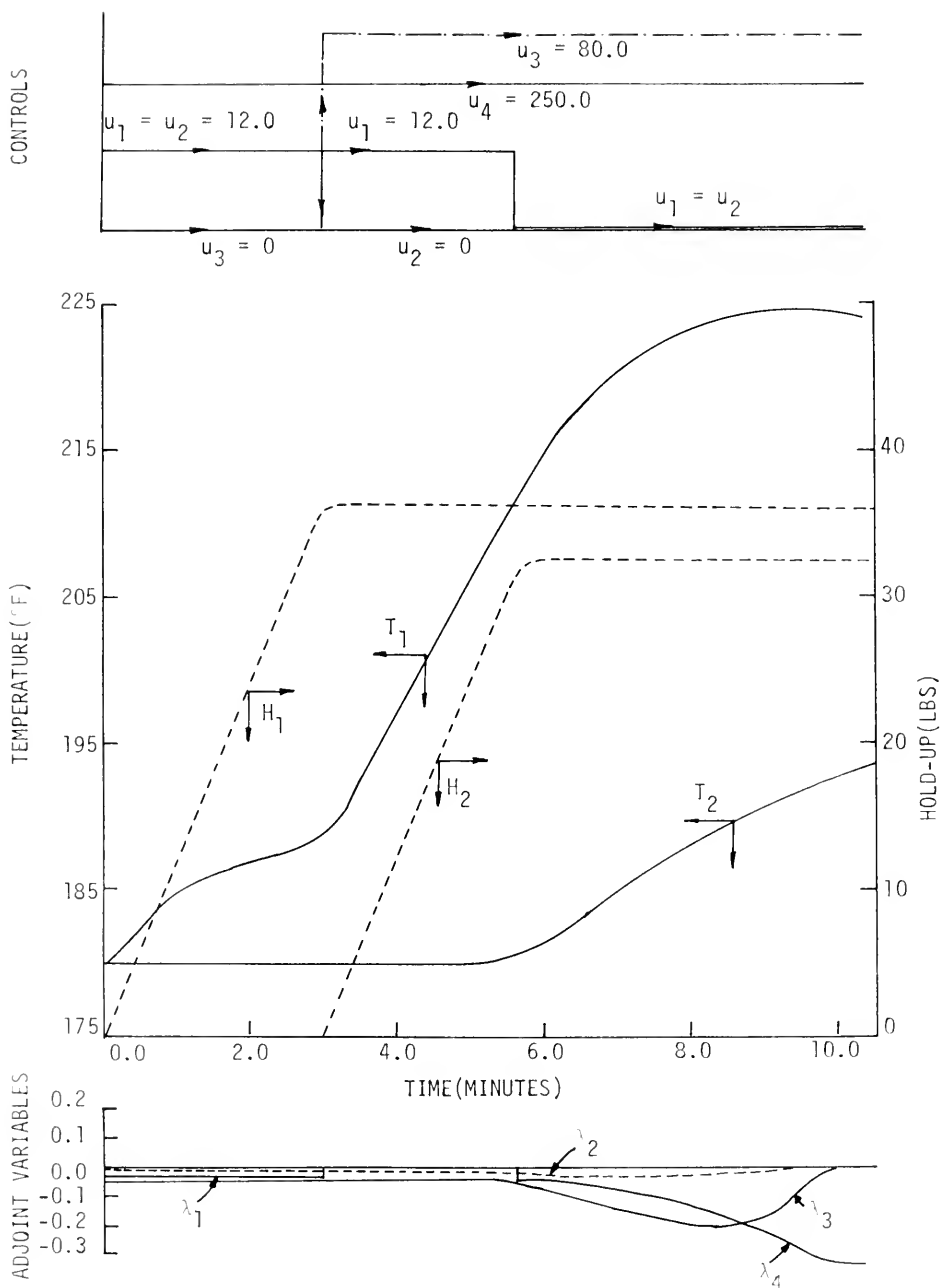


Figure 4.6 Control, State and Adjoint Variables for Problem 3, Iteration 1

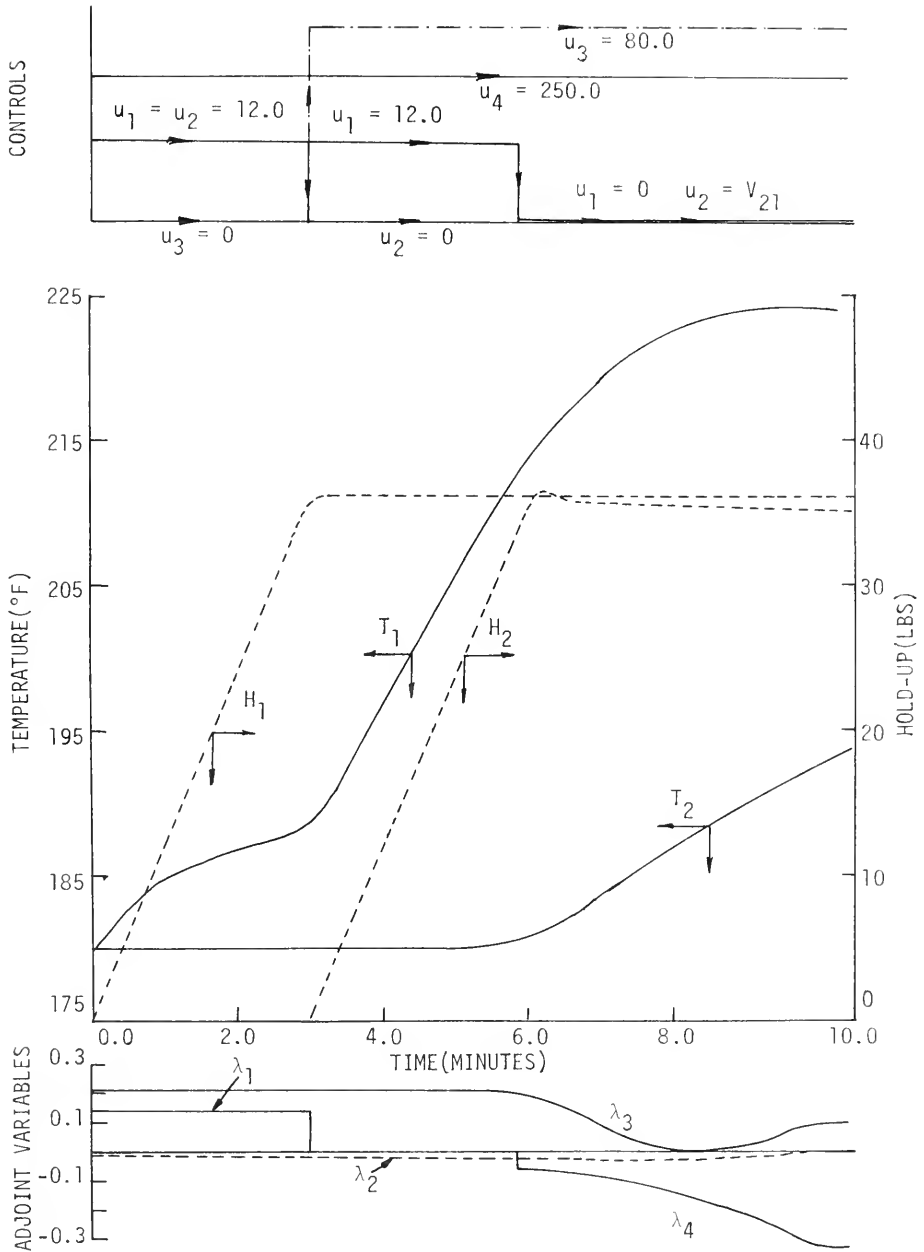


Figure 4.7 Control, State and Adjoint Variables for Problem 3, Iteration 3

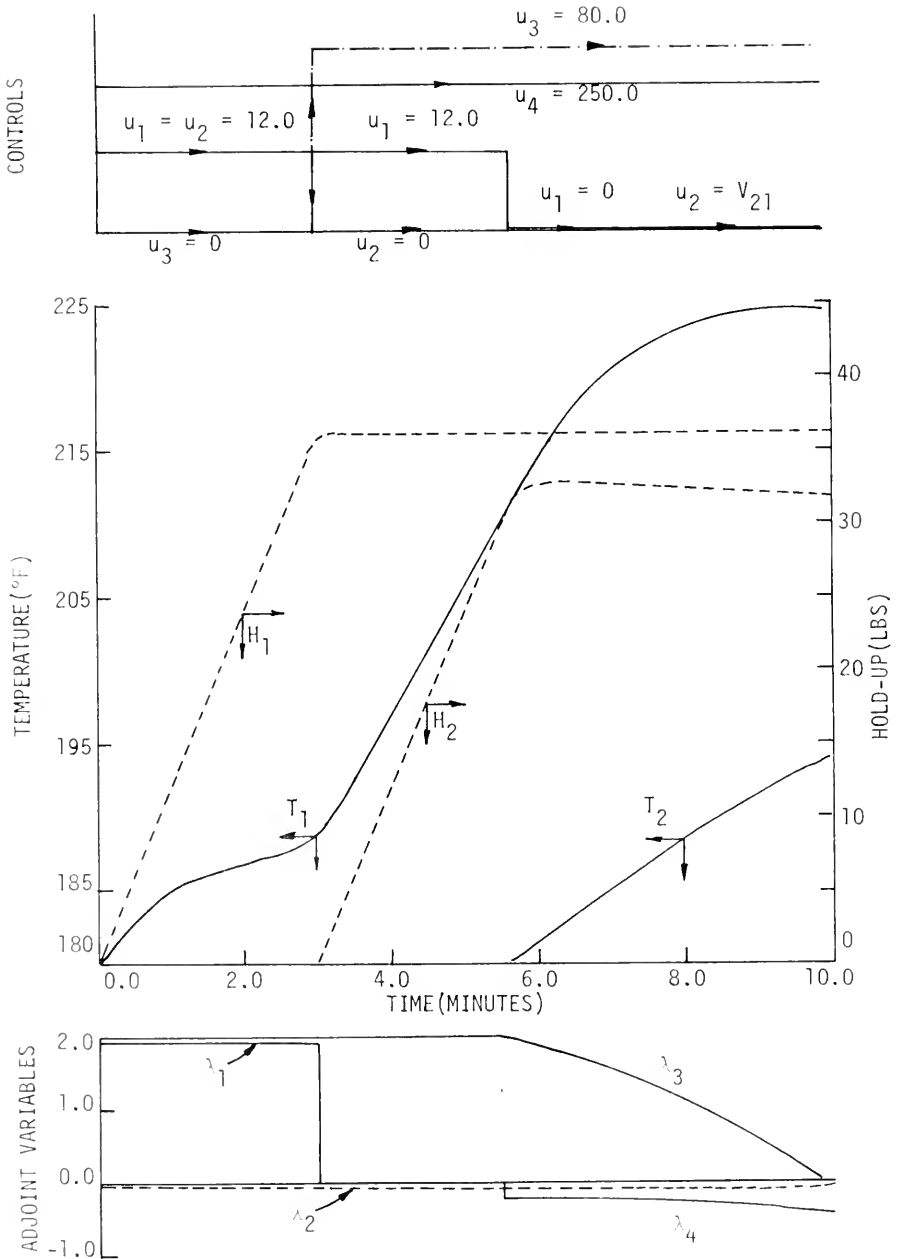


Figure 4.8 Control, State and Adjoint Variables for Problem 3, Iteration 5

TABLE 4.11

ADJOINT VARIABLES FOR PROBLEM 3, ITERATION 1

TIME (MINS)	A D J O I N T		V A R I A B L E S		C O N T R O L		V A R I A B L E S	
	LAMBD A1	LAMBD A2	LAMBD A3	LAMBD A4	U1	U2	U3	U4
					(L B S / M I N)		(D E G F)	
10.38	0.0	0.0	0.0	-0.3300	12.00	0.45	87.00	250.00
9.88	0.0	-0.0135	-0.1042	-0.2727	12.00	0.44	87.00	250.00
9.25	0.0	-0.0185	-0.0140	-0.2483	12.00	0.11	87.00	250.00
8.25	0.0	-0.0310	-0.2048	-0.1539	12.00	0.11	87.00	250.00
7.50	0.0	-0.0309	-0.1779	-0.1082	12.00	0.12	87.00	250.00
6.63	0.0	-0.0257	-0.1177	-0.0725	12.00	0.11	87.00	250.00
6.38	0.0	-0.0237	0.0592	-0.0647	12.00	0.11	87.00	250.00
5.63	0.0	-0.0172	-0.0483	-0.0461	12.00	0.09	87.00	250.00
5.13	0.0	-0.0156	-0.0456	0.0	12.00	0.0	87.00	250.00
4.63	0.0	-0.0144	-0.0456	0.0	12.00	0.0	87.00	250.00
4.00	0.0	-0.0130	-0.0456	0.0	12.00	0.0	87.00	250.00
3.69	0.0	-0.0124	-0.0456	0.0	12.00	0.0	87.00	250.00
3.31	0.0	-0.0117	-0.0456	0.0	12.00	0.0	87.00	250.00
2.50	-0.0349	-0.0094	-0.0456	0.0	12.00	12.00	87.00	250.00
1.50	-0.0311	-0.0055	-0.0456	0.0	12.00	12.00	87.00	250.00
0.50	-0.0261	-0.0017	-0.0456	0.0	12.00	12.00	87.00	250.00

TABLE 4.12

STATE VARIABLES FOR PROBLEM 3, ITERATION 3

TIME (MINS)	S T A T E V A R I A B L E S				C O N T R O L V A R I A B L E S			
	X1 (LBS)	X2 (BTU/LB)	X3 (LBS)	X4 (BTU/LB)	U1 (L B S / M I N)	U2	U3	U4 (DEG F)
0.0	0.0	147.92	0.0	147.92	12.00	12.00	0.0	250.00
1.00	12.00	152.91	0.06	147.92	12.00	12.00	0.0	250.00
2.75	33.00	155.42	0.07	147.92	12.00	12.00	0.0	250.00
3.09	35.82	156.68	1.37	147.92	12.00	0.0	80.00	250.00
3.50	36.18	160.48	5.88	147.92	12.00	0.0	80.00	250.00
3.88	36.18	164.08	10.39	147.92	12.00	0.0	80.00	250.00
4.38	36.18	168.57	16.39	147.92	12.00	0.0	80.00	250.00
4.88	36.18	172.71	22.39	147.92	12.00	0.0	80.00	250.00
5.38	36.18	176.53	28.39	147.92	12.00	0.0	80.00	250.00
6.13	36.18	182.12	36.30	148.96	0.0	0.10	80.00	250.00
6.50	36.18	184.61	35.85	150.07	0.0	0.11	80.00	250.00
6.75	36.18	186.05	35.86	150.81	0.0	0.11	80.00	250.00
7.00	36.18	187.32	35.83	151.58	0.0	0.11	80.00	250.00
7.25	36.18	188.42	35.80	152.36	0.0	0.12	80.00	250.00
7.75	36.18	190.19	35.74	153.95	0.0	0.12	80.00	250.00
8.25	36.18	191.59	35.68	155.53	0.0	0.12	80.00	250.00
9.25	36.18	193.64	35.56	158.66	0.0	0.11	80.00	250.00
10.25	36.18	193.50	35.30	161.15	0.0	0.10	80.00	250.00
10.50	36.18	192.26	35.12	161.39	0.0	0.43	80.00	250.00
10.75	36.18	191.64	34.96	161.55	0.0	0.41	80.00	250.00
11.25	36.18	189.97	34.57	161.54	0.0	0.37	80.00	250.00
11.75	36.18	189.59	34.26	161.71	0.0	0.36	80.00	250.00
12.25	36.18	188.85	33.92	161.71	0.0	0.34	80.00	250.00
12.75	36.18	188.39	33.59	161.71	0.0	0.33	80.00	250.00
13.25	36.18	188.19	33.27	161.71	0.0	0.32	80.00	250.00
13.75	36.18	188.07	32.95	161.71	0.0	0.32	80.00	250.00
14.75	36.18	187.91	32.32	161.71	0.0	0.31	80.00	250.00

TABLE 4.13

ADJOINT VARIABLES FOR PROBLEM 3, ITERATION 3

TIME (MINS)	A D J O I N T LAMBDA1	V A R I A B L E S LAMBDA2	V A R I A B L E S LAMBDA3	V A R I A B L E S LAMBDA4	C O N T R O L U1	V A R I A B L E S U2 (L B S / M I N)	V A R I A B L E S U3	V A R I A B L E S U4 (D E G F)
10.38	0.0	0.0	0.0900	-0.3300	0.0	0.11	80.00	250.00
9.25	0.0	-0.0171	0.0159	-0.2594	0.0	0.11	80.00	250.00
8.54	0.0	-0.0276	0.0072	-0.1902	0.0	0.12	80.00	250.00
8.00	0.0	-0.0302	0.0336	-0.1503	0.0	0.11	80.00	250.00
6.88	0.0	-0.0266	0.1283	-0.0934	0.0	0.11	80.00	250.00
6.50	0.0	-0.0239	0.1624	-0.0796	0.0	0.11	80.00	250.00
5.88	0.0	-0.0188	0.2141	-0.0615	0.0	0.10	80.00	250.00
4.88	0.0	-0.0147	0.2387	0.0	0.0	0.0	80.00	250.00
4.25	0.0	-0.0133	0.2387	0.0	0.0	0.0	80.00	250.00
3.88	0.0	-0.0125	0.2387	0.0	0.0	0.0	80.00	250.00
3.31	0.0	-0.0115	0.2387	0.0	0.0	0.0	80.00	250.00
2.79	0.1304	-0.0104	0.2387	0.0	12.00	12.00	0.0	250.00
1.50	0.1349	-0.0054	0.2387	0.0	12.00	12.00	0.0	250.00
0.00	0.1388	-0.0002	0.2387	0.0	12.00	12.00	0.0	250.00

TABLE 4.14
STATE VARIABLES FOR PROBLEM 3, ITERATION 5

TIME (MINS)	S T A T E X1 (LBS)	V A R X2 (BTU/LB)	I A B L E S X3 (LBS)	X4 (BTU/LB)	C O N T R O L U1 (L B S / M I N)	U2 S / M I N	V A R I A B L E S U3 (DEG F)	U4 (DEG F)
0.0	0.0	147.92	0.0	147.92	12.00	12.00	0.0	250.00
0.50	6.00	150.34	0.07	147.92	12.00	12.00	0.0	250.00
1.00	12.00	152.91	0.06	147.92	12.00	12.00	0.0	250.00
1.50	18.00	154.17	0.07	147.92	12.00	12.00	0.0	250.00
2.00	24.00	154.72	0.07	147.92	12.00	12.00	0.0	250.00
2.50	30.00	155.26	0.07	147.92	12.00	12.00	0.0	250.00
2.75	33.00	155.42	0.07	147.92	12.00	12.00	0.0	250.00
2.88	34.50	155.49	0.07	147.92	12.00	12.00	0.0	250.00
3.09	35.82	156.68	1.37	147.92	12.00	0.0	80.00	250.00
3.31	36.18	158.61	3.64	147.92	12.00	0.0	80.00	250.00
3.50	36.18	160.48	5.88	147.92	12.00	0.0	80.00	250.00
3.69	36.18	162.31	8.14	147.92	12.00	0.0	80.00	250.00
4.00	36.18	165.24	11.89	147.92	12.00	0.0	80.00	250.00
4.25	36.18	167.48	14.89	147.92	12.00	0.0	80.00	250.00
4.85	36.18	172.71	22.39	147.92	12.00	0.0	80.00	250.00
5.38	36.18	176.53	28.39	147.92	12.00	0.0	80.00	250.00
5.84	36.18	181.12	32.86	148.89	0.0	0.10	80.00	250.00
6.25	36.18	184.27	32.80	150.14	0.0	0.11	80.00	250.00
6.50	36.18	185.81	32.77	150.94	0.0	0.11	80.00	250.00
6.75	36.18	187.12	32.74	151.77	0.0	0.11	80.00	250.00
7.00	36.18	188.24	32.72	152.61	0.0	0.11	80.00	250.00
7.50	36.18	190.08	32.66	154.31	0.0	0.12	80.00	250.00
8.25	36.18	192.13	32.57	156.87	0.0	0.11	80.00	250.00
8.75	36.18	193.19	32.51	158.84	0.0	0.11	80.00	250.00
9.25	36.18	194.08	32.46	160.18	0.0	0.11	80.00	250.00
9.75	36.18	193.26	32.25	161.23	0.0	0.10	80.00	250.00
9.88	36.18	192.66	32.18	161.46	0.0	0.44	80.00	250.00

TABLE 4.15
ADJOINT VARIABLES FOR PROBLEM 3, ITERATION 5

TIME (MINS)	A D J O I N T LAMBDA1	LAMBDA2	V A R I A B L E S LAMBDA3	LAMBDA4	C O N T R O L U1	U2 (L B S / M I N)	V A R I A B L E S U3	U4 (D E G F)
9.88	0.0	0.0	0.0050	-0.3460	0.0	0.11	87.00	250.00
9.50	0.0	-0.0247	0.3613	-0.3192	0.0	0.11	87.00	250.00
8.00	0.0	-0.0503	1.0908	-0.2718	0.0	0.12	87.00	250.00
7.25	0.0	-0.0528	1.4226	-0.2518	0.0	0.12	87.00	250.00
6.75	0.0	-0.0516	1.6267	-0.2395	0.0	0.11	80.00	250.00
6.33	0.0	-0.0493	1.7666	-0.2309	0.0	0.11	87.00	250.00
5.88	0.0	-0.0448	1.9351	-0.2200	0.0	0.09	87.00	250.00
5.13	0.0	-0.0392	2.0010	0.0	0.0	0.0	87.00	250.00
4.38	0.0	-0.0347	2.0010	0.0	0.0	0.0	87.00	250.00
4.00	0.0	-0.0327	2.0010	0.0	0.0	0.0	87.00	250.00
3.50	0.0	-0.0303	2.0010	0.0	0.0	0.0	87.00	250.00
2.88	1.9774	-0.0265	2.0000	0.0	12.00	12.00	0.0	250.00
2.00	1.9842	-0.0184	2.0010	0.0	12.00	12.00	0.0	250.00
0.50	2.0069	-0.0042	2.0000	0.0	12.00	12.00	0.0	250.00

in a shorter final time but at the expense of a larger deviation of the second effect hold-up, x_3 , from the steady state value, \hat{x}_3 .

The optimal policy for this problem can be put in a feedback form by getting rid of the dependence of the feed to the second effect, u_1 , on the switching time. It should be noted that the switching time for u_1 in Problem 1 was approximately 5.66 minutes which is close to that for Problem 2. This is not unusual as the switch from $u_{1,\max}$ to $u_{1,\min}$ in Problem 2 occurred when the excess hold-up in the second effect was just sufficient to drain out by the time the second effect liquid started to boil. As the vaporization in the first effect is of the order of 0.1 lbs per minute (0.0008 kg/s), the second effect should be overfilled by about 0.5 lbs (0.23 kg), above the final hold-up of 32 lbs (14.56 kg) to compensate for the vaporization in the first effect. The major difference between the optimal policy of Problem 3 and that of Problem 1 is that the second effect feed flow rate is stopped after the second effect fills up whereas $u_1 = u_2$, the intereffect feed flow rate, in Problem 1 during the same stage of startup.

Summary of the control policy:

Feed flow, $u_2 = u_{2,\max}$ for $x_1 < \hat{x}_1$ (first effect filling)

Feed flow, $u_2 = v_{21}$ for $x_1 \geq \hat{x}_1$ (first effect filled)

Intereffect liquid flow, $u_1 = u_{1,\max}$ for $x_3 < \hat{x}_3$ (second effect filling)

Intereffect liquid flow, $u_1 = u_{1,\min}$ for $x_3 \geq \hat{x}_3$ (second effect filled)

Recirculation rate, $u_3 = 0$ for $x_1 < \hat{x}_1$ (first effect filling)

Recirculation rate, $u_3 = u_{3,\max}$ for $x_1 \geq \hat{x}_1$ (first effect filled)

Steam temperature, $u_4 = u_{4,\max}$ for all time.

IV.4 Experimental Runs

Start-up runs were made on the evaporator as a final test of the minimum time policy with a bound on the second effect hold-up (as in Problem 1, Section IV.3.1). These runs consequently served as a test for the evaporator model as well. The procedure for each run was as follows:

- 1) The feed to the second effect was preheated to around 180°F.
- 2) The temperature in the steam chest of the first effect, u_4 , was kept at the maximum possible value throughout the run.
- 3) The feed rates to the two effects were at their maximum values initially. The feed to the first effect was cut off as soon as the first effect hold-up reached its steady state value. This hold-up, x_1 , was then put on analog control by manipulation of the feed rate to the first effect, u_2 , by one of the automatic controllers.
- 4) The recirculation pump was started and the recirculation flow rate, u_3 , rapidly built up to its maximum value.
- 5) The feed rate to the second effect, u_1 , was kept at its maximum value until the second effect hold-up, x_3 , reached its steady state value. This hold-up was then put on analog control by manipulation of the feed flow rate to the second effect, u_1 .
- 6) The flow rates, temperatures, hold-ups and vacuum pressure were sampled automatically by the IBM 1070 unit every 15 to 20 seconds. The 1070 unit also maintained the recirculation rate u_3 constant by moving the setpoint of the controller which operated on the recirculation valve CV3 (see Figure 2.2) depending upon the deviation

of the measured flow u_3 from the desired flow. A linear Kalman filter was built for this particular flow as it was subject to a lot of pulsations and noise. The discrete filter was formulated as follows:

$$F_n = F_{n-1} + v_{n-1} \quad (4.26)$$

$$Y_n = F_n + w_n$$

where F_n is the flow rate u_3 which we wish to maintain constant. Y_n is the measured value of the flow. v and w are random noise sequences of mean zero and covariances q and r , respectively. The filter equations were obtained from Jazwinski (1970).

Given $\hat{F}_{n/n}$ and $P_{n/n}$, where $\hat{F}_{n/n}$ is the estimate of F and $P_{n/n}$ is the covariance of the estimate, we can predict the flow rate and covariance to the next sampling time by

$$\hat{F}_{n+1/n} = \hat{F}_{n/n}$$

and

$$P_{n+1/n} = P_{n/n} + q_n$$

On sampling the flow rate y_{n+1} at the next sampling time, we can update the covariance and estimate

$$P_{n+1/n+1} = P_{n+1/n} - \frac{p_{n+1/n}^2}{p_{n+1/n} + r_{n+1}}$$

$$\hat{F}_{n+1/n+1} = \hat{F}_{n+1/n} + \frac{P_{n+1/n+1}}{r_{n+1}} (y_{n+1} - \hat{F}_{n+1/n})$$

This was the recursive scheme used to obtain an estimate of the inter-effect flow rate, u_3 . It was found that regardless of what the initial value for the covariance P_0 was the estimate converged rapidly--generally

within three or four sampling points. To allow for this convergence to the real flow rate, control was not done at the first few sampling points.

Figure 4.9 shows the behavior of the filtered flow rate u_3 as opposed to the measured flow rate. In all three cases shown it can be seen that the filtered estimate is representative of the actual flow after about 6 sampling intervals regardless of the initial estimate of the variance of the estimate p . In most of the runs an initial estimate of unity was taken for p . The variances of the process and measurement noise were also taken as unity.

The results of complete runs through start-up are shown in Figures 4.10 to 4.16. In these figures the experimental minimum time, t_{exp} , is compared with either the theoretical minimum time predicted by the model, t_{opt} , using idealized controls or the minimum time predicted by the model, t_{act} , using the actual controls measured in the experiment. The initial and final conditions for the model simulations were the same as those of the experimental runs.

Figure 4.10 is a comparison of the experimental and actual minimum times for Run C1. The data and simulated values are listed in Tables 4.16 and 4.18. The control variable values used for the simulation are the smoothed values obtained in the experiment. Note that in this run and in the remaining runs the hold-ups measured are not quite near those predicted. Some possible explanations for this discrepancy are given in Chapter V. Figure 4.11 compares the experimental time with the theoretical minimum time, t_{opt} , using idealized controls--in other words, the control variables used on the

simulation were shut off and on precisely. Tabulated values for this simulation can be found in Table 4.17.

Figure 4.12 shows the results of experimental run C2 and the simulation using the actual controls. As these controls were more realistic than the idealized controls, all subsequent simulations used the actual controls. Data and simulated values for this run are listed in Tables 4.19 and 4.20.

Run C3 was a start-up run in which the recirculation rate was held at around 100 lbs/min as opposed to 80 lbs/min for runs C1 and C2. The experimental and simulated values are plotted in Figure 4.13 and listed in Tables 4.21 and 4.22. A similar recirculation rate was used in run C4 and these results can be found in Figure 4.14 and in Tables 4.23 and 4.24.

Runs C5 and C6 were different from runs C1 to C4 in that the recirculation rate was maintained at 65 lbs/min. Figures 4.15 and 4.16 along with Tables 4.25 to 4.28 contain the experimental and simulated values for runs C5 and C6.

It can be seen from Figures 4.10 to 4.16 that the model is fairly representative of the process. There is about a 15 percent discrepancy in the prediction of the final time by the model to the final time obtained experimentally. This could be either due to deficiencies in the model or in the experimental setup. The next chapter lists some of these deficiencies and a few suggestions are made to overcome them.

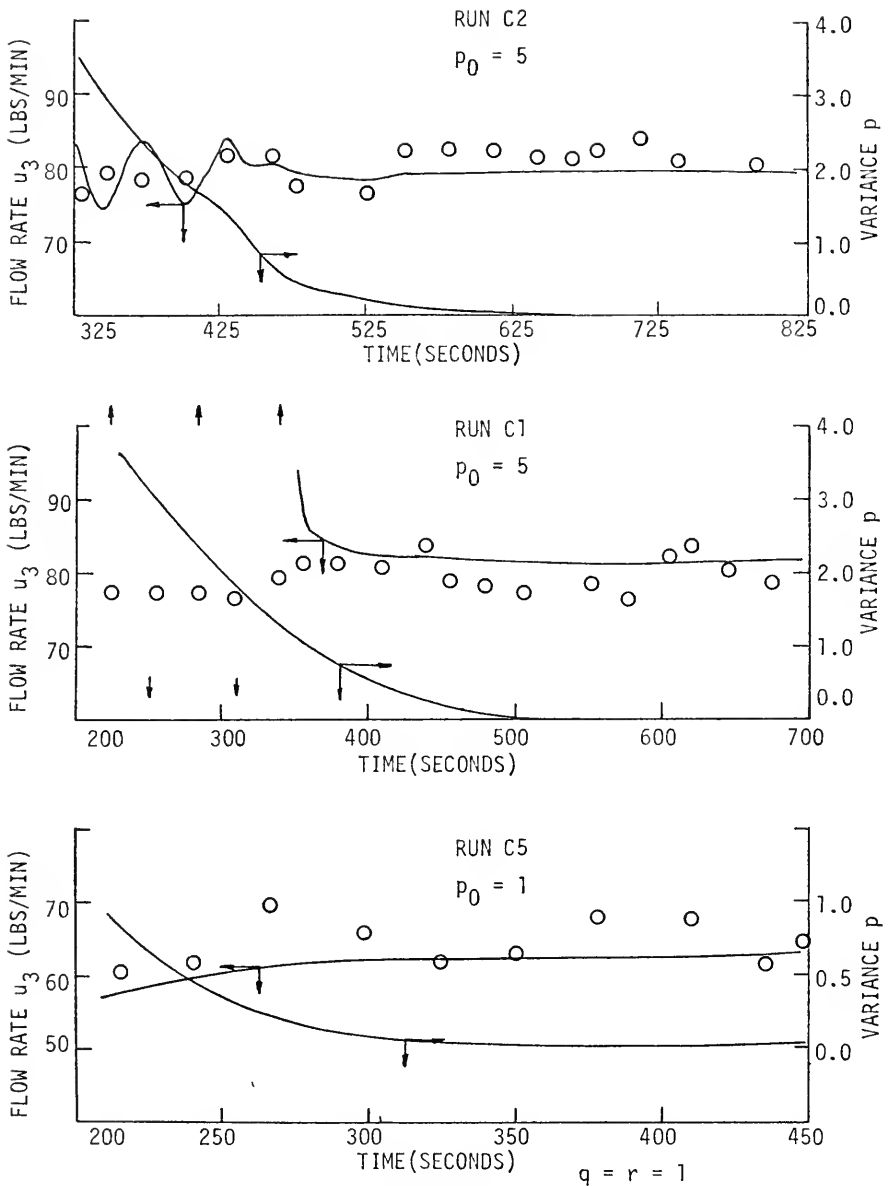


Figure 4.9 Filtered versus Actual Flow Rate

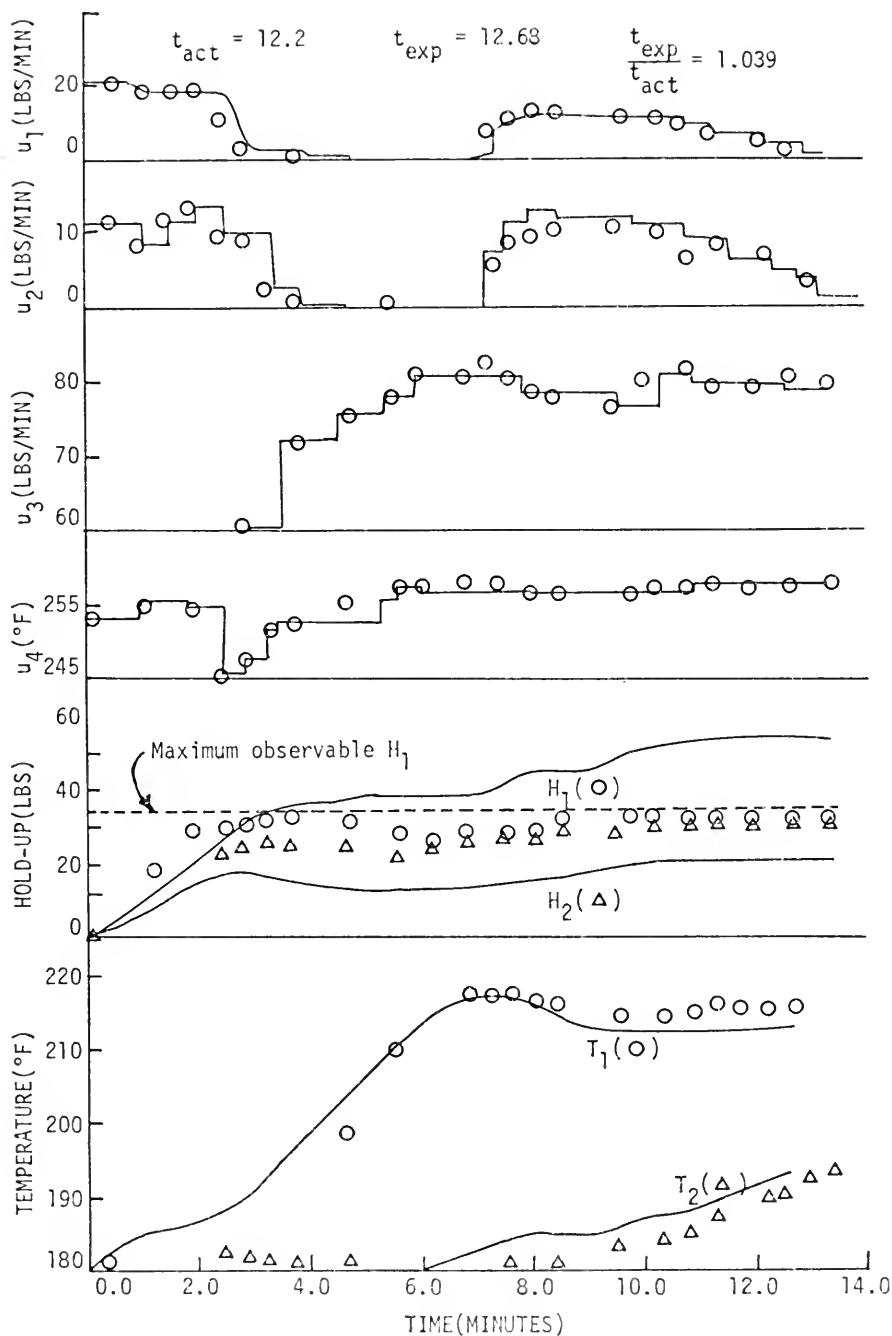


Figure 4.10 Experimental versus Actual Minimum Time for Run C1

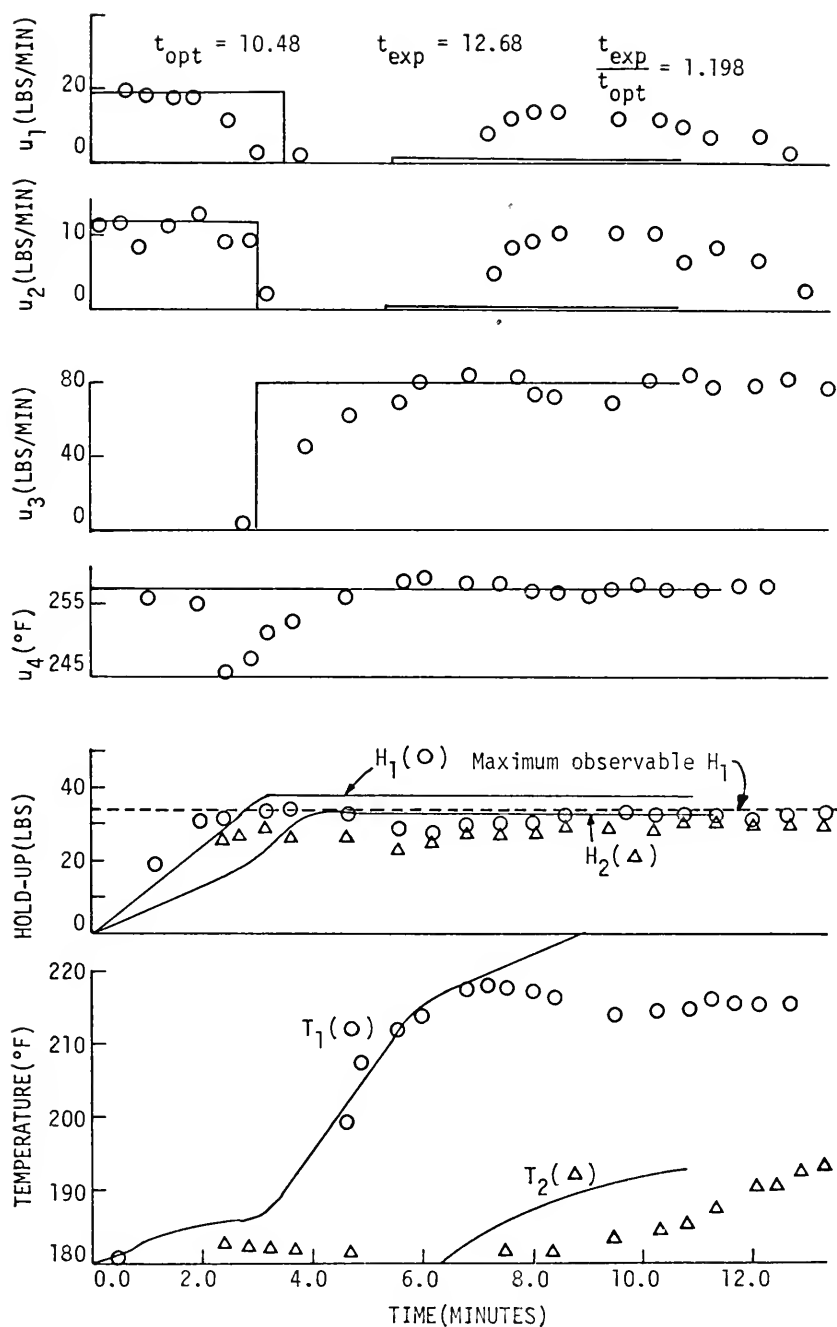


Figure 4.11 Experimental versus Optimal Minimum Time for Run C1

TABLE 4.16
EXPERIMENTAL DATA FOR RUN C1

TIME (MINS)	FLOW WF	RATES W12P	IN LBS/MIN W11	W11F	HOLD-UPS(LBS) H1 H2	TEMPERATURES IN DEG F TF Y12 T2	COV P
0.06	20.23	10.97	0.0		4.3 20.6	175.8 167.8 179.4 253.9	
0.48	19.94	11.96	0.0		1.7 21.5	176.7 168.7 179.4 252.3	
0.95	19.68	7.53	0.0		8.5 24.8	176.7 170.9 179.4 256.4	
1.42	19.51	13.20	0.0		10.4 28.2	176.3 171.4 179.4 255.1	
1.89	19.35	14.17	0.0		9.3 31.0	176.7 170.5 179.8 254.7	
2.41	7.60	8.45	69.36		33.0 31.6	177.3 124.0 183.0 240.8	
2.83	0.0	9.43	64.98		32.2 31.6	177.3 152.5 182.6 248.7	
3.33	0.0	0.0	58.61	37.68	33.5 32.2	176.8 167.9 182.1 253.2	4.29
3.76	0.0	0.0	77.54	181.32	33.1 32.8	176.7 175.4 181.6 253.1	3.60
4.76	0.0	0.0	77.15	356.11	32.4 31.1	176.9 199.8 181.3 256.9	2.36
5.64	0.0	0.0	79.12	159.51	32.0 29.6	176.9 211.4 179.1 257.8	1.35
5.98	0.0	0.0	81.40	85.82	32.1 29.4	177.3 213.0 178.6 258.5	0.94
6.82	0.0	0.0	80.53	82.17	31.7 26.4	177.3 217.7 178.2 258.9	0.39
7.32	10.93	7.90	83.57	82.48	31.4 27.8	177.2 216.4 179.9 257.2	0.22
7.58	12.71	10.13	79.24	82.08	31.3 28.4	177.2 216.8 181.2 256.8	0.12
7.99	13.67	10.05	78.27	81.83	32.0 29.4	177.2 215.5 180.8 256.4	0.17
8.46	11.96	10.72	77.75	81.70	31.9 28.8	177.4 215.3 181.4 256.6	0.03
9.63	10.71	10.83	76.25	81.59	32.0 29.4	177.9 213.3 183.7 256.7	0.01
10.37	10.24	10.00	83.41	81.59	33.1 29.7	177.5 213.7 184.6 257.5	0.0
10.81	8.36	5.48	80.41	81.60	33.5 31.5	177.5 215.0 185.0 257.5	0.0
11.28	5.46	9.12	78.60	81.59	33.4 31.5	177.5 215.4 187.7 257.9	0.0
12.23	4.35	6.24	78.87	81.59	33.8 31.5	176.5 216.2 190.2 257.8	0.0
12.45	3.83	0.0	79.87	81.59	32.8 31.5	177.0 216.6 190.7 257.8	0.0
12.92	0.0	3.48	78.11	81.59	32.5 31.7	177.0 217.9 192.9 257.8	0.0
13.45	0.0	0.0	78.97	81.59	32.0 30.1	176.6 218.4 193.8 257.9	0.0
13.86	0.0	0.0	79.33	81.59	32.2 29.5	177.1 218.4 193.4 257.9	0.0

TABLE 4.17

THEORETICAL MINIMUM TIME SIMULATION FOR RUN C1

TIME (MINS)	S T A T E		V A R I A B L E S		C O N T R O L		V A R I A B L E S	
	(LBS)	(BTU/LB)	(LBS)	(BTU/LB)	(LBS)	(LBS / MIN)	(BTU/LB)	(DEG F)
0.0	0.0	145.50	0.0	145.41	19.00	12.00	0.0	257.00
0.50	6.00	148.24	2.26	145.41	19.00	12.00	0.0	257.00
1.00	12.00	151.06	5.90	145.41	19.00	12.00	0.0	257.00
1.50	18.00	152.53	9.26	145.41	19.00	12.00	0.0	257.00
1.75	21.00	152.84	11.00	145.41	19.00	12.00	0.0	257.00
2.00	24.00	153.17	12.75	145.41	19.00	12.00	0.0	257.00
2.25	27.00	153.43	14.50	145.41	19.00	12.00	0.0	257.00
2.50	30.00	153.65	16.25	145.41	19.00	12.00	0.0	257.00
2.75	33.00	153.82	18.01	145.41	19.00	12.00	0.0	257.00
3.23	37.44	155.28	22.77	145.41	19.00	0.0	80.00	257.00
3.81	37.44	161.66	31.82	145.41	0.0	0.0	80.00	257.00
4.00	37.44	163.61	32.42	145.41	0.0	0.0	80.00	257.00
4.31	37.44	166.75	33.17	145.41	0.0	0.0	80.00	257.00
4.56	37.44	169.15	33.23	145.41	0.0	0.0	80.00	257.00
4.81	37.44	171.47	33.18	145.41	0.0	0.0	80.00	257.00
5.06	37.44	173.69	33.17	145.41	0.0	0.0	80.00	257.00
5.31	37.44	175.84	33.17	145.41	0.0	0.0	80.00	257.00
5.69	37.44	179.66	33.17	145.67	0.11	0.11	80.00	257.00
6.19	37.44	186.25	33.17	147.45	0.13	0.13	80.00	257.00
6.69	37.44	190.20	33.17	149.41	0.14	0.14	80.00	257.00
7.19	37.44	192.97	33.17	151.43	0.14	0.14	80.00	257.00
7.69	37.44	195.04	33.17	153.46	0.14	0.14	80.00	257.00
8.44	37.44	197.36	33.17	156.48	0.14	0.14	80.00	257.00
9.44	37.44	197.31	33.17	158.10	0.13	0.13	80.00	257.00
9.81	37.44	197.93	33.17	159.10	0.13	0.13	80.00	257.00
10.19	37.43	197.42	33.17	158.96	0.13	0.13	80.00	257.00
10.44	37.43	197.73	33.17	159.46	1.50	0.62	80.00	257.00

TABLE 4.18

ACTUAL MINIMUM TIME SIMULATION FOR RUN C1

TIME (MINS)	S T A T E		V A R I A B L E S		C O N T R O L			V A R I A B L E S	
	X1	X2	X3	X4	U1	U2	U3	U4	
	(LBS)	(BTU/LB)	(LBS)	(BTU/LB)	(LBS / MIN)			(DEG F)	
0.06	0.0	147.39	0.0	147.39	20.23	10.97	0.0	253.92	
0.56	6.01	149.77	2.39	147.39	20.03	11.66	0.0	252.77	
1.56	15.78	154.24	12.42	147.39	19.59	11.87	0.0	255.18	
1.68	17.20	154.44	13.45	147.39	19.59	11.87	0.0	255.18	
1.93	20.26	154.82	15.28	147.39	19.42	13.48	0.0	254.87	
2.06	22.00	154.95	15.96	147.39	19.42	13.48	0.0	254.87	
2.31	25.35	155.17	17.47	147.39	19.42	13.48	0.0	254.87	
2.50	27.44	156.18	18.01	147.39	11.15	9.96	48.55	245.05	
2.81	30.58	158.15	18.41	147.39	11.15	9.96	48.55	245.05	
2.93	31.81	158.90	18.24	147.39	3.34	9.59	60.05	247.60	
3.18	34.20	160.47	16.57	147.39	3.34	9.59	60.05	247.60	
3.31	35.40	161.15	15.74	147.39	3.34	9.59	60.05	247.60	
3.68	37.06	163.63	14.68	147.39	1.00	2.88	59.04	251.52	
3.87	37.40	165.06	14.46	147.39	0.30	0.86	71.99	252.63	
4.13	37.68	167.59	14.28	147.39	0.30	0.86	71.99	252.63	
4.31	37.79	168.57	14.21	147.39	0.30	0.86	71.99	252.63	
4.56	38.01	170.46	14.07	147.39	0.30	0.86	71.99	252.63	
4.68	38.12	171.37	13.99	147.39	0.30	0.86	71.99	252.63	
5.93	38.45	180.63	13.78	147.39	0.03	0.08	78.06	257.13	
6.68	38.43	183.63	13.76	150.33	0.01	0.02	80.40	258.08	
7.43	38.87	185.87	13.94	153.24	7.65	5.53	82.65	257.64	
8.00	43.47	184.35	15.36	153.14	12.93	9.66	78.87	256.60	
8.56	48.91	182.67	17.14	152.70	12.25	10.40	78.09	256.58	
9.56	59.28	180.30	18.96	153.72	12.25	10.40	78.09	256.58	
10.56	69.89	179.96	19.59	155.33	10.52	10.21	81.43	257.22	
11.56	77.76	180.22	19.86	158.13	6.52	8.45	79.23	257.73	
12.56	85.12	180.48	18.38	163.15	4.18	2.07	79.60	257.82	

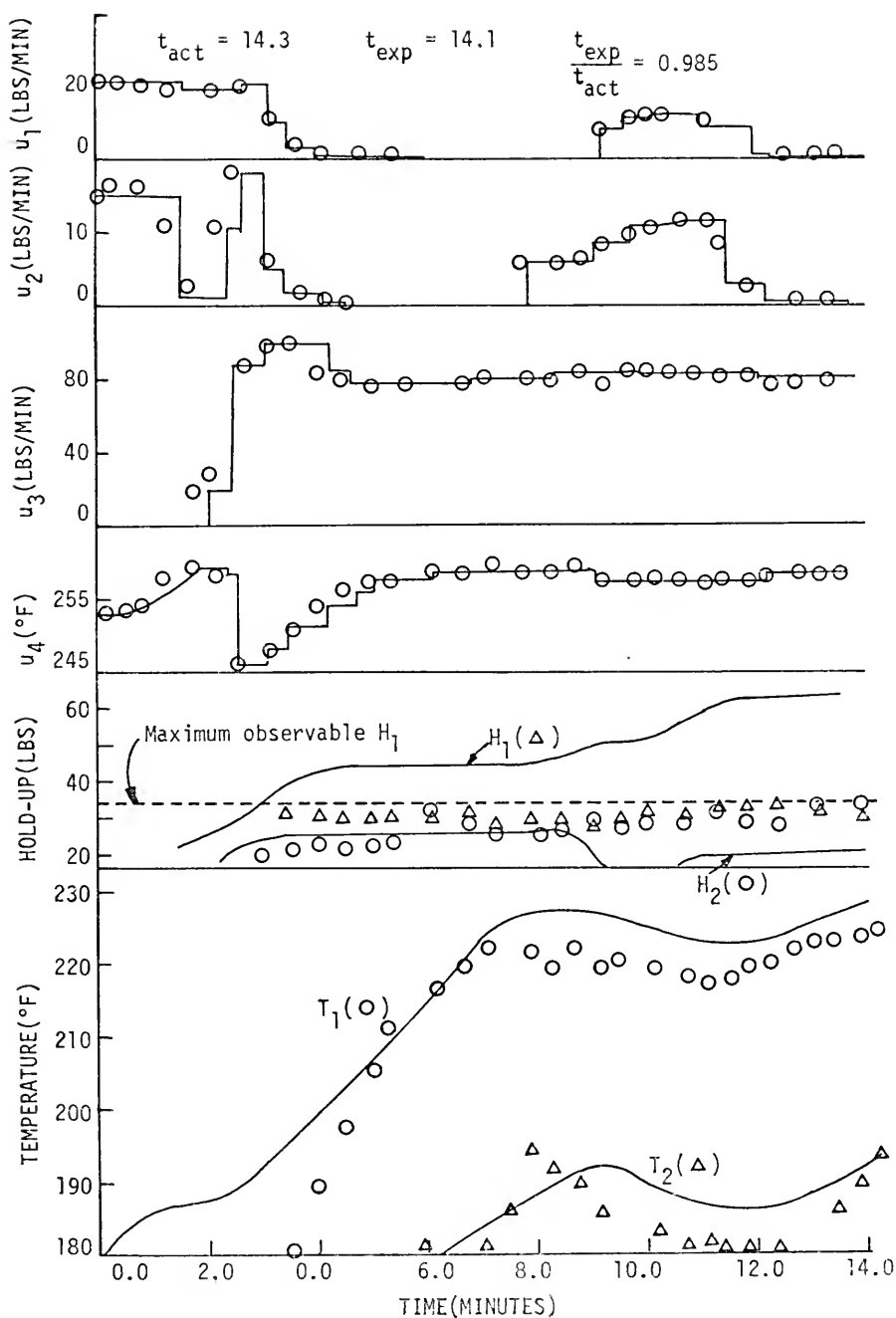


Figure 4.12 Experimental versus Actual Minimum Time for Run C2

TABLE 4.19
EXPERIMENTAL DATA FOR RUN C2

TIME (MINS)	FLOW WF	RATES W12P	IN LBS/MIN W11	W11F	HOLD-UPS(LBS) H1 H2	TEMPERATURES IN DEG F TF T12 T2 TS	COV P
0.06	21.25	14.93	0.0		1.7 20.3	174.4 164.6 178.0 253.0	
0.32	19.98	17.21	0.0		10.3 20.5	174.4 165.5 178.4 253.5	
0.81	20.33	15.68	0.0		11.1 20.5	174.0 169.1 178.4 254.7	
1.32	20.24	7.07	0.0		9.1 19.8	174.4 170.0 178.4 258.8	
1.73	19.56	0.0	27.40		23.4 21.6	174.4 163.7 178.4 260.8	
2.20	20.01	14.30	32.77		26.2 27.7	174.0 163.5 178.4 257.5	
2.67	20.72	21.27	109.50		31.6 29.2	174.4 138.6 178.0 240.6	
3.14	6.79	0.53	105.83		32.5 32.6	174.0 167.3 178.0 248.9	
3.67	0.0	0.0	99.49		33.4 33.8	174.0 179.8 177.6 252.3	
4.08	0.0	0.0	78.17		33.6 33.8	173.6 189.9 177.1 255.5	
4.55	0.0	0.0	77.81		33.1 33.5	173.6 198.7 177.1 257.2	
5.01	0.0	0.0	77.06	74.60	32.5 33.5	173.1 205.1 176.7 257.5	4.29
5.48	0.0	0.0	76.79	82.50	32.2 33.2	173.6 211.2 176.7 257.9	3.60
6.19	0.0	0.0	78.24	83.73	31.1 31.2	173.5 216.7 176.6 259.1	2.36
6.67	0.0	0.0	78.66	75.04	30.8 31.2	173.5 219.7 176.6 259.1	1.82
7.18	0.0	0.0	81.93	84.31	31.2 33.4	173.5 222.2 177.5 259.1	1.85
7.83	0.0	8.52	81.20	80.87	30.6 30.3	173.5 220.9 194.7 258.7	0.62
8.35	0.0	6.11	77.46	79.56	30.1 24.5	172.7 219.7 191.7 258.8	0.39
8.76	0.0	6.32	79.83	78.95	29.6 26.3	172.7 221.8 189.1 259.2	0.22
9.23	11.32	9.42	82.32	75.36	28.7 25.7	172.7 218.9 185.1 257.2	0.12
9.70	12.70	10.43	82.36	79.56	29.4 28.0	172.7 220.1 182.9 257.1	0.07
10.17	13.46	10.79	82.61	79.66	29.8 28.9	172.7 218.8 181.1 257.2	0.03
10.70	12.01	12.34	81.83	79.71	31.2 28.1	172.6 217.6 181.1 257.1	0.02
11.11	11.33	11.49	81.19	79.71	30.0 28.1	173.1 217.1 180.6 257.1	0.01
11.87	0.0	0.0	84.13	79.73	30.9 32.5	173.0 218.8 180.1 257.5	0.0
12.75	0.0	0.0	80.10	79.73	30.8 32.9	173.0 221.3 176.7 258.7	0.0
13.54	0.0	0.0	78.34	79.73	29.7 29.0	172.9 222.9 185.8 258.2	0.0
14.40	0.0	0.0	80.10	79.73	30.3 31.3	173.9 224.7 193.3 259.1	0.0

TABLE 4.20

ACTUAL MINIMUM TIME SIMULATION FOR RUN C2

TIME (MINS)	S T A T E			V A R I A B L E S			C O N T R O L			V A R I A B L E S		
	X1 (LBS)	X2 (BTU/LB)	X3 (BTU/LB)	X4 (BTU/LB)	U1 (L B S / M I N)	U2 (L B S / M I N)	U3 (DEG F)	U4 (DEG F)				
0.06	0.0	142.45	0.0	146.01	21.25	14.93	0.7	253.05				
1.56	22.88	152.66	6.68	146.01	20.27	9.73	0.0	257.41				
2.18	25.14	155.75	16.84	146.01	19.77	2.92	19.18	259.76				
2.81	32.35	159.22	22.16	146.01	20.49	18.15	85.54	245.91				
3.31	40.28	161.61	23.56	146.01	10.90	5.82	99.74	248.00				
3.87	42.59	165.97	25.66	146.01	3.27	1.75	99.57	250.99				
4.37	43.13	169.94	26.12	146.01	0.98	0.52	84.59	254.17				
4.75	43.24	172.79	26.22	146.01	0.29	0.16	79.84	256.27				
5.12	43.30	175.52	26.27	146.01	0.09	0.05	77.90	257.16				
5.87	43.30	181.29	26.28	146.36	0.03	0.01	77.12	257.71				
6.62	43.21	188.97	26.29	149.88	0.01	0.00	77.90	258.69				
7.37	43.12	193.70	26.29	153.51	0.00	0.00	80.88	259.07				
8.06	44.87	194.96	24.44	156.92	0.00	5.96	81.10	258.81				
8.50	47.43	194.75	21.84	158.94	0.00	6.07	78.55	258.80				
8.87	49.68	194.90	19.56	160.10	0.0	6.24	79.45	259.08				
9.25	52.11	195.04	17.46	160.65	7.92	8.47	81.46	257.74				
9.62	55.32	194.50	17.37	159.33	7.92	8.47	81.46	257.74				
9.87	57.69	194.01	17.62	158.25	11.27	9.84	82.09	257.31				
10.12	60.13	193.51	17.96	157.28	11.27	9.84	82.09	257.31				
10.50	64.00	192.71	18.76	155.91	12.80	10.51	82.45	257.22				
10.87	68.19	191.85	19.25	154.93	12.25	11.79	82.02	257.15				
11.62	76.22	190.71	19.51	153.99	9.09	8.41	82.13	257.41				
12.12	78.63	191.46	19.69	154.47	2.73	2.52	83.53	257.44				
12.62	79.29	192.88	19.75	155.55	0.82	0.76	81.90	258.02				
13.12	79.47	194.38	19.77	156.84	0.25	0.23	80.64	258.50				
13.75	79.48	196.07	19.77	158.55	0.02	0.02	78.55	258.20				
14.50	79.92	197.57	20.16	160.08	19.94	10.89	28.69	258.20				

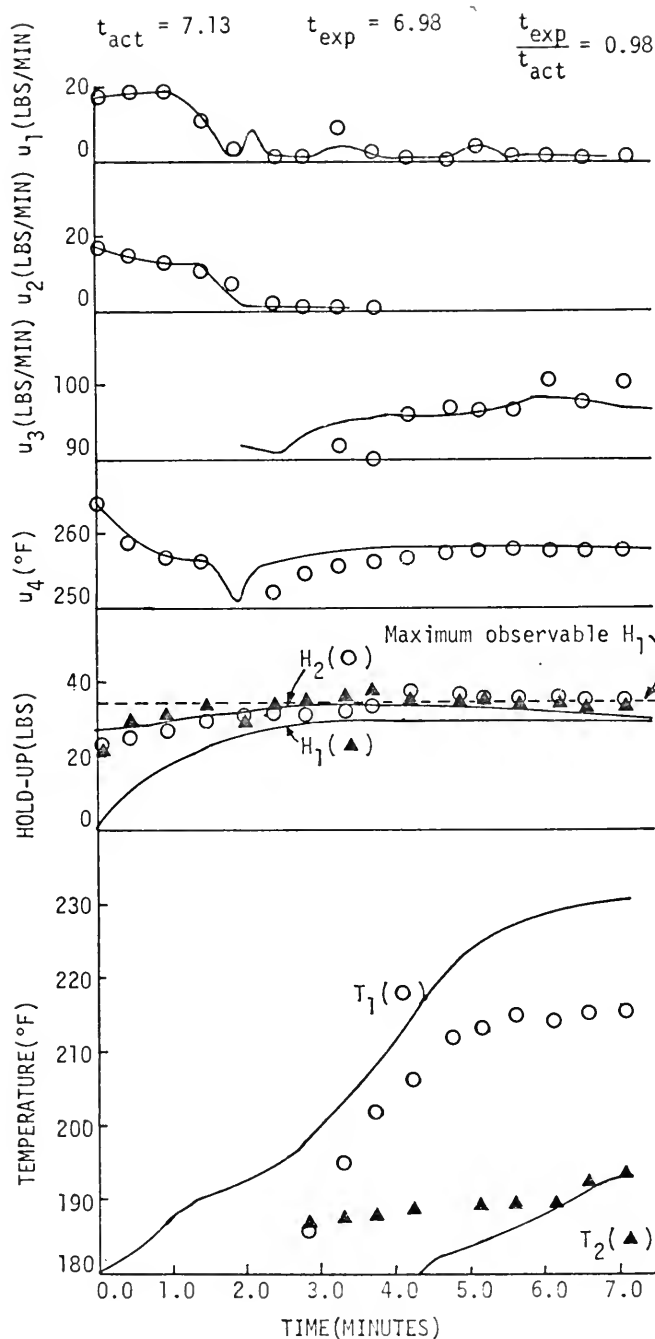


Figure 4.13 Experimental versus Actual Minimum Time for Run C3

TABLE 4.21

EXPERIMENTAL DATA FOR RUN C3

TIME (MINS)	FLOW WF	RATES W12P	IN LBS/MIN W11	HOLD-UPS(LBS) W11F	H1	H2	TEMPERATURES IN DEG F T1 T2	TS	COV P
0.07	17.68	17.38	0.0		20.7	25.7	182.7 190.5	180.5 264.2	
0.48	18.16	13.37	0.0		26.6	27.9	182.7 167.4	183.1 255.7	
0.95	18.51	12.40	17.17		27.8	30.9	182.7 158.1	184.9 255.7	
1.42	7.55	10.84	32.77		30.9	32.7	182.8 161.8	188.1 255.8	
1.89	0.0	4.39	0.0		31.9	32.9	182.3 147.8	191.1 245.5	
2.42	0.0	0.0	84.98		32.7	34.5	182.4 175.8	190.3 253.4	
2.83	0.0	0.0	93.72		32.6	34.2	182.4 185.5	189.5 255.0	
3.30	11.65	0.0	94.76		32.0	33.6	182.4 194.7	189.0 255.4	
3.78	0.0	0.0	88.70	90.72	31.9	32.4	182.5 201.3	188.7 256.3	0.66
4.25	0.0	0.0	97.85	93.69	31.5	31.8	183.0 205.2	188.2 256.8	0.41
4.78	0.0	0.0	98.36	94.83	31.5	32.1	183.5 209.1	188.7 257.2	0.24
5.19	4.20	0.0	96.92	95.12	31.9	31.9	183.0 210.0	189.6 257.6	0.13
5.66	0.0	0.0	96.88	95.24	31.5	31.7	183.0 210.8	196.6 257.6	0.07
6.13	2.26	0.0	102.19	95.50	31.6	30.7	182.6 210.5	196.6 257.3	0.03
6.60	0.0	0.0	98.36	95.56	31.5	31.0	182.6 210.9	197.1 257.3	0.01
7.13	0.0	0.0	102.53	95.62	31.8	32.7	183.1 211.8	197.6 257.7	0.0
7.55	0.0	0.0	98.74	95.64	31.9	32.4	183.1 211.8	197.6 257.7	0.0
8.02	6.06	4.14	98.86	95.64	31.9	30.7	182.7 212.3	198.0 257.7	0.0
8.35	0.0	3.78	97.20	95.65	31.4	31.3	182.8 212.4	197.7 257.4	0.0
8.72	0.0	3.94	98.89	95.65	31.9	31.0	182.8 212.4	198.1 257.4	0.0
9.20	0.0	2.95	99.30	95.65	31.3	31.3	182.8 213.2	198.6 257.9	0.0

TABLE 4.22

ACTUAL MINIMUM TIME SIMULATION FOR RUN C3

TIME (MINS)	S T A T E (LBS)	X1 (LBS)	X2 (BTU/LB)	X3 (LBS)	X4 (BTU/LB)	C O N T R O L U1	U2 (L B S / M I N)	V A R I A B L E S U3	U4 (DEG F)
0.07	0.0	0.0	148.47	25.70	148.47	17.68	17.38	0.0	264.19
0.57	7.91	7.91	151.48	27.91	148.32	18.02	14.57	0.0	258.24
1.07	14.79	14.79	156.34	30.88	148.21	18.36	13.05	12.02	256.45
1.57	20.95	20.95	158.79	32.74	148.15	10.79	11.50	26.54	255.97
1.76	23.11	23.11	159.82	32.74	148.13	3.24	6.52	7.96	248.66
1.94	25.05	25.05	160.53	32.87	148.12	0.97	1.96	61.88	251.99
2.07	25.71	25.71	160.58	32.87	148.12	0.29	0.59	84.17	254.12
2.32	27.39	27.39	160.87	32.87	148.11	2.47	0.05	89.56	255.96
2.73	28.58	28.58	164.19	34.53	148.11	0.22	0.00	95.71	256.60
2.91	28.81	28.81	166.25	34.23	148.11	0.07	0.00	93.69	256.71
3.29	29.03	29.03	170.64	34.23	148.11	0.01	0.00	94.83	257.15
3.41	29.09	29.09	172.07	33.62	148.11	2.94	0.00	96.29	257.46
3.66	29.13	29.13	175.00	33.62	148.10	1.15	0.0	96.35	257.59
4.10	29.13	29.13	180.56	32.45	148.37	2.09	0.0	95.50	257.28
4.48	29.09	29.09	186.24	31.81	149.62	0.19	0.0	95.56	257.25
4.79	29.06	29.06	189.55	32.06	150.48	0.02	0.0	95.62	257.68
5.04	29.02	29.02	191.67	32.06	151.80	0.00	0.0	95.64	257.72
5.16	29.01	29.01	192.58	32.06	152.27	4.24	2.10	97.89	257.72
5.29	28.99	28.99	196.13	31.73	154.59	5.51	3.77	95.64	257.72
6.04	28.88	28.88	197.18	31.73	155.68	0.15	3.89	97.92	257.46
6.54	28.81	28.81	198.81	30.67	157.62	0.01	3.24	98.20	257.74
6.79	28.78	28.78	199.47	30.96	158.61	0.00	3.04	95.65	257.82
7.04	28.74	28.74	200.05	30.96	159.70	0.0	0.0	98.21	257.74
7.29	28.71	28.71	200.59	32.74	160.71	0.0	0.0	97.98	257.53
7.48	28.67	28.67	200.68	32.74	161.37	0.01	0.00	95.65	257.71

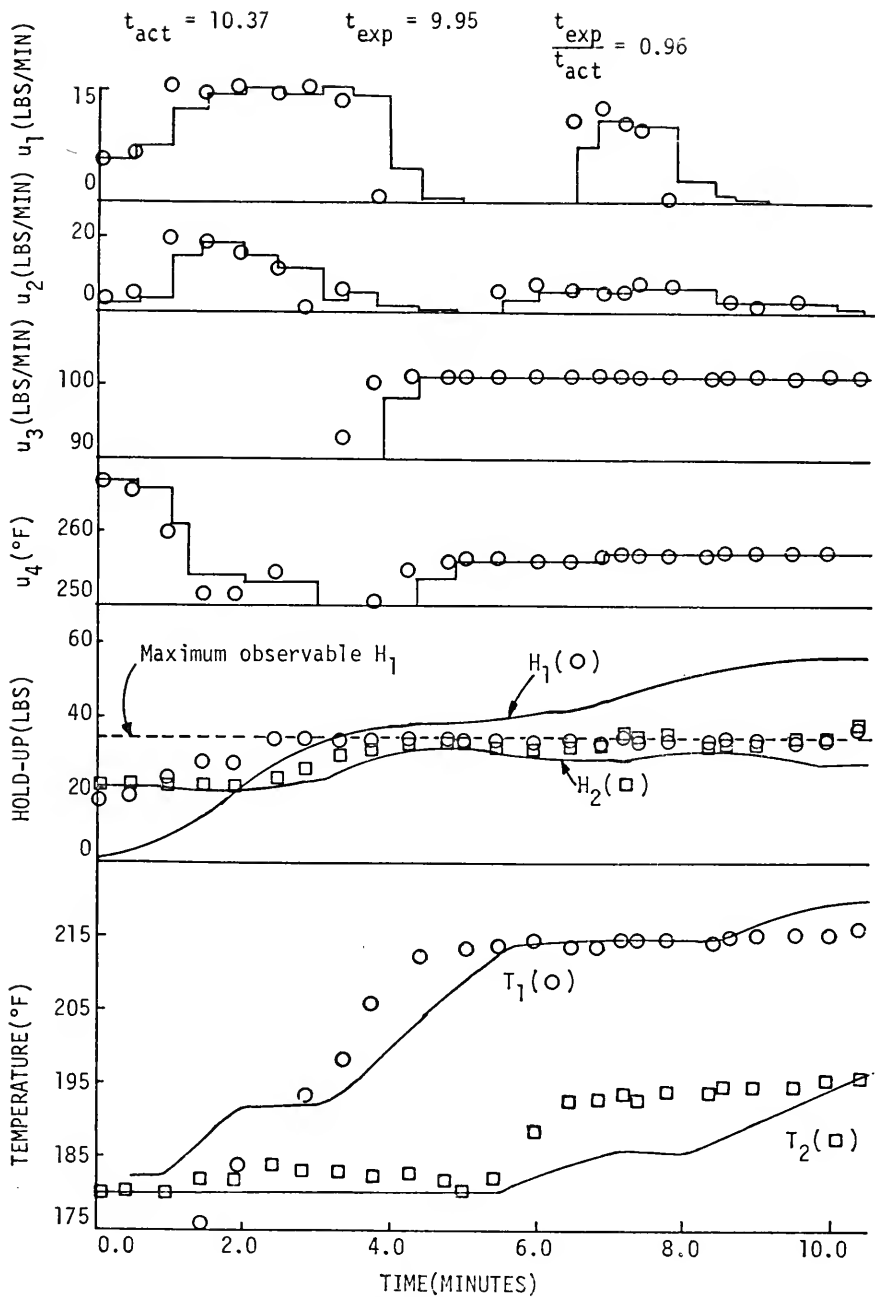


Figure 4.14 Experimental versus Actual minimum Time for Run C4

TABLE 4.23
EXPERIMENTAL DATA FOR RUN C4

TIME (MINS)	FLOW		RATES		IN LBS/MIN		HOLD-UPS(LBS)		TEMPERATURES IN DEG F				COV P
	WF	W12P	W11	W11F	H1	H2	TF	T12	T2	TS			
0.06	5.63	3.00	28.00		17.7	20.7	179.6	175.2	180.0	267.4			
0.48	6.24	4.25	35.32		18.5	21.3	179.6	170.0	180.4	265.4			
0.95	15.51	19.12	37.15		23.4	21.6	179.1	177.1	180.0	259.8			
1.42	14.75	19.36	0.0		27.2	21.5	179.6	182.5	181.8	251.6			
1.89	15.64	14.98	0.0		26.6	21.6	180.0	185.0	181.8	251.6			
2.42	14.54	10.88	0.0		33.6	22.9	179.6	186.6	183.1	254.5			
2.83	15.32	0.0	117.91		33.5	25.0	179.6	187.4	182.7	239.6			
3.30	13.79	6.24	92.40		32.9	29.4	179.6	188.9	182.7	247.5			
3.77	0.0	0.0	100.40		33.4	33.2	179.8	195.4	182.0	250.5			
4.25	0.0	0.0	101.55		33.3	33.2	180.2	203.7	182.4	254.2			
4.77	0.0	0.0	101.02	101.11	33.5	32.6	180.2	203.5	181.5	255.8		0.41	
4.99	0.0	0.0	100.69	101.01	33.1	32.3	179.8	205.4	180.2	256.3		0.24	
5.42	0.0	4.62	100.90	100.95	32.9	32.1	179.8	208.6	182.0	256.3		0.13	
5.98	0.0	6.26	102.97	101.13	32.6	31.9	179.3	210.3	188.1	255.8		0.07	
6.42	10.34	6.45	101.80	101.16	33.0	32.0	179.7	210.5	192.5	255.8		0.03	
6.83	12.87	4.72	99.49	101.13	32.7	31.9	179.7	212.0	192.5	256.2		0.01	
7.16	10.65	5.94	103.20	101.15	33.2	33.5	179.8	211.8	193.4	256.7		0.0	
7.36	9.53	6.70	99.90	101.14	33.1	33.8	179.8	211.7	193.0	256.7		0.0	
7.78	0.0	6.24	100.62	101.14	33.1	33.8	179.8	212.1	193.8	257.1		0.0	
8.31	0.0	0.0	99.49	101.14	33.3	32.4	179.5	211.6	193.5	256.8		0.0	
8.51	0.0	2.63	99.20	101.14	33.1	32.3	179.5	211.4	194.4	257.2		0.0	
8.96	0.0	1.73	98.44	101.14	33.2	32.3	179.9	212.8	194.4	257.2		0.0	
9.48	0.0	2.39	99.20	101.13	33.3	31.9	179.8	212.9	194.3	257.1		0.0	
9.91	0.0	0.0	99.07	101.13	32.9	32.6	179.8	213.5	195.2	257.1		0.0	
10.38	0.0	0.0	97.68	101.13	34.8	35.9	179.8	213.1	195.6	225.6		0.0	

TABLE 4.24

ACTUAL MINIMUM TIME SIMULATION FOR RUN C4

TIME (MINS)	S T A T E V A R I A B L E S				C O N T R O L V A R I A B L E S			
	X1 (LBS)	X2 (BTU/LB)	X3 (LBS)	X4 (BTU/LB)	U1 (L B S / M I N)	U2	U3	U4 (DEG F)
0.06	0.0	133.00	20.00	148.00	5.63	3.00	28.00	267.43
0.31	0.71	135.78	20.46	147.06	5.63	3.00	28.00	267.43
1.06	4.72	152.67	21.69	147.18	12.67	14.55	35.94	261.64
1.62	13.64	159.47	20.22	147.91	14.13	17.92	10.78	254.61
2.56	28.39	158.76	19.32	147.95	14.73	12.37	0.97	253.87
2.81	31.52	158.63	19.88	147.91	14.73	12.37	0.97	253.87
3.25	33.94	161.62	24.04	147.91	15.14	3.71	82.83	243.92
3.75	36.63	165.44	28.48	147.92	14.20	5.48	89.53	246.42
4.12	37.58	168.55	30.00	147.92	4.26	1.64	97.14	249.31
4.50	37.90	171.94	30.50	147.92	0.38	0.15	101.17	253.78
4.75	37.95	174.26	30.58	147.92	0.38	0.15	101.17	253.78
5.12	37.97	177.70	30.61	147.92	0.03	0.01	101.01	255.95
5.68	38.97	180.72	29.62	149.08	0.01	3.24	100.99	256.17
6.00	40.04	181.34	28.54	150.87	0.00	5.35	101.13	255.92
6.37	42.08	181.69	26.49	152.93	0.00	5.35	101.13	255.92
6.75	44.30	182.02	26.47	152.84	7.24	6.12	101.16	255.84
7.00	45.63	182.25	27.76	152.50	11.18	5.14	101.13	256.11
7.37	47.70	182.46	29.71	152.45	9.91	6.40	101.14	256.62
7.75	50.08	182.46	31.01	152.62	9.91	6.40	101.14	256.62
8.00	51.65	182.50	30.53	153.27	2.97	6.29	101.14	256.94
8.37	53.80	182.70	29.48	154.42	0.89	1.89	101.14	256.84
8.75	54.50	183.61	28.92	156.15	0.27	2.41	101.14	257.10
9.12	55.29	184.48	28.18	157.97	0.08	1.93	101.14	257.18
9.50	56.02	185.38	27.47	159.73	0.02	2.25	101.13	257.13
9.75	56.58	185.96	26.91	160.86	0.02	2.25	101.13	257.13
10.00	57.07	186.56	26.42	161.92	0.01	0.68	101.13	257.12
10.25	57.21	187.30	26.27	162.97	0.01	0.68	101.13	257.12

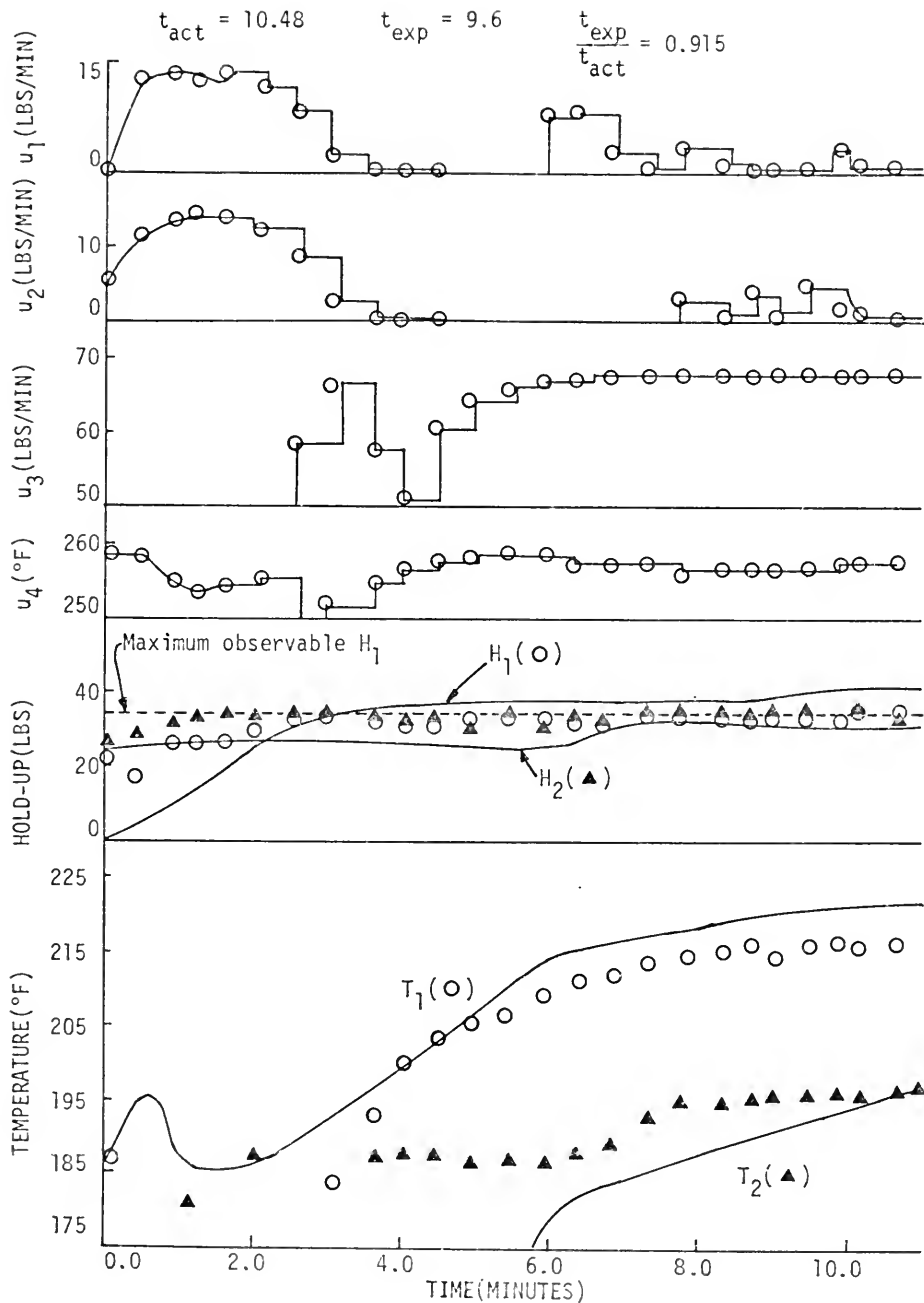


Figure 4.15 Experimental versus Actual Minimum Time for Run C5

TABLE 4.25

EXPERIMENTAL DATA FOR RUN C5

TIME (MINS)	FLOW WF	RATES W12P	IN LBS/MIN W11	W11F	HOLD-UPS(LBS) H1 H2	TEMPERATURES IN DEG F TF T12 T2 TS	COV P
0.06	19.32	24.96	0.0		19.2 19.5	180.6 117.1 181.9 264.8	
0.48	18.85	17.53	0.0		27.4 19.5	181.0 140.4 183.2 255.8	
0.95	18.92	19.36	0.0		28.4 19.5	181.0 160.9 184.1 253.4	
1.42	19.16	0.0	55.71		32.2 19.3	181.4 153.2 185.4 249.2	
1.89	18.70	0.0	68.03		31.2 26.2	181.4 163.6 185.4 248.4	
2.41	0.0	0.0	64.92	65.96	31.1 29.5	181.4 178.8 185.0 252.9	0.66
2.87	0.0	0.0	62.51	64.52	31.2 29.3	181.4 186.7 185.0 254.5	0.41
3.30	0.0	0.0	68.46	65.48	31.1 27.6	181.4 199.4 184.5 255.8	0.24
3.77	0.0	0.0	61.65	64.96	30.9 28.7	181.4 203.3 184.1 256.6	0.07
4.24	0.0	0.0	62.54	64.79	30.4 28.4	181.4 206.7 183.1 256.6	0.03
4.76	0.0	0.0	68.34	64.92	30.5 27.1	181.4 206.7 181.2 256.6	0.01
4.99	0.0	0.0	64.53	64.91	30.6 27.1	181.4 208.5 180.4 257.0	0.0
5.41	0.0	2.69	66.18	64.93	30.4 27.1	181.3 209.2 181.3 257.3	0.0
5.96	5.20	0.76	66.31	64.93	30.4 27.3	181.3 210.1 181.4 256.5	0.0
6.35	7.13	3.10	65.80	64.93	30.8 27.9	181.3 210.1 182.6 256.9	0.0
6.82	9.93	2.19	66.93	64.94	30.6 27.9	181.4 210.6 183.2 257.0	0.0
7.29	11.91	5.17	67.47	64.94	30.4 27.4	181.9 211.5 184.1 257.4	0.0
7.57	10.81	5.66	61.57	64.94	30.0 27.6	181.4 212.8 185.4 257.4	0.0
8.03	7.27	0.0	65.39	64.94	30.7 27.8	181.8 213.9 185.4 257.3	0.0
8.47	4.11	4.21	68.94	64.94	30.7 29.5	181.8 213.5 185.5 257.3	0.0
8.94	5.18	4.62	63.63	64.94	31.1 29.5	180.8 214.3 185.0 256.9	0.0
9.47	2.97	2.68	63.08	64.95	30.8 31.0	180.8 214.7 184.9 257.3	0.0
9.88	0.46	2.84	64.21	64.94	31.3 30.7	181.3 214.3 184.5 257.3	0.0
10.35	0.0	3.86	65.07	64.94	31.1 31.0	181.7 215.6 191.2 258.1	0.0
10.69	0.0	0.0	64.28	64.94	30.8 29.3	181.7 216.4 193.3 257.7	0.0
11.06	0.0	2.69	64.34	64.94	31.0 29.3	181.7 215.6 194.7 258.1	0.0
11.58	4.54	0.0	64.28	64.94	31.0 29.3	181.8 218.7 195.8 228.8	0.0
12.47	0.0	0.0	63.21	64.94	33.3 35.6		

TABLE 4.26

ACTUAL MINIMUM TIME SIMULATION FOR RUN C5													
TIME (MINS)	S T A T E		V A R I A B L E S		C O N T R O L		V A R I A B L E S		U3		U4		
	X1 (LBS)	X2 (BTU/LB)	X3 (LBS)	X4 (BTU/LB)	U1 (L B S / M I N)	U2	U3	U4 (DEG F)					
0.06	0.0	149.90	0.0	149.90	19.32	24.96	0.0	264.75					
0.56	12.05	155.08	0.0	149.90	18.99	19.76	0.0	258.51					
1.06	21.68	156.23	0.0	149.90	18.94	19.48	0.0	254.92					
1.37	27.77	156.69	0.0	149.90	18.94	19.48	0.0	254.92					
1.84	30.96	159.70	3.58	147.82	19.09	5.84	39.00	250.94					
2.25	31.93	162.98	10.27	147.89	18.82	1.75	59.32	249.17					
2.56	32.36	165.55	14.48	147.90	5.65	0.53	63.24	251.79					
2.93	32.51	168.80	15.98	147.90	1.69	0.16	62.73	253.72					
3.25	32.56	171.46	16.48	147.90	1.69	0.16	62.73	253.72					
3.50	32.58	173.60	16.65	147.90	0.51	0.05	66.74	254.59					
3.75	32.59	175.68	16.78	147.90	0.51	0.05	66.74	254.59					
4.00	32.60	177.66	16.83	147.90	0.15	0.01	63.18	255.44					
4.25	32.60	179.56	16.85	147.90	0.05	0.00	62.73	256.27					
4.75	32.59	186.26	16.89	148.68	0.05	0.00	62.73	256.27					
5.25	32.57	191.29	16.90	149.99	0.00	0.00	65.17	256.56					
5.75	33.30	194.08	16.14	151.47	0.00	1.88	65.88	256.87					
6.25	33.86	196.38	17.02	152.65	3.64	1.10	66.18	257.18					
7.00	35.49	197.49	19.70	153.64	8.78	2.28	66.63	256.86					
7.50	36.76	197.55	23.06	154.55	10.97	4.30	67.22	256.96					
8.25	40.24	196.03	27.55	156.12	8.35	1.58	64.75	257.37					
8.50	40.74	196.23	29.02	156.67	5.38	3.42	67.68	257.33					
8.87	41.99	196.23	29.66	157.66	5.38	3.42	67.68	257.33					
9.25	43.48	196.05	30.10	158.52	5.24	4.26	64.85	257.32					
9.50	44.47	195.98	30.32	159.06	3.65	3.15	63.51	257.01					
10.12	46.30	196.33	30.22	160.72	1.42	2.93	64.03	257.20					
10.37	47.04	196.49	29.79	161.50	0.43	3.58	64.76	257.25					
10.62	47.91	196.55	28.98	162.37	0.43	3.58	64.76	257.25					

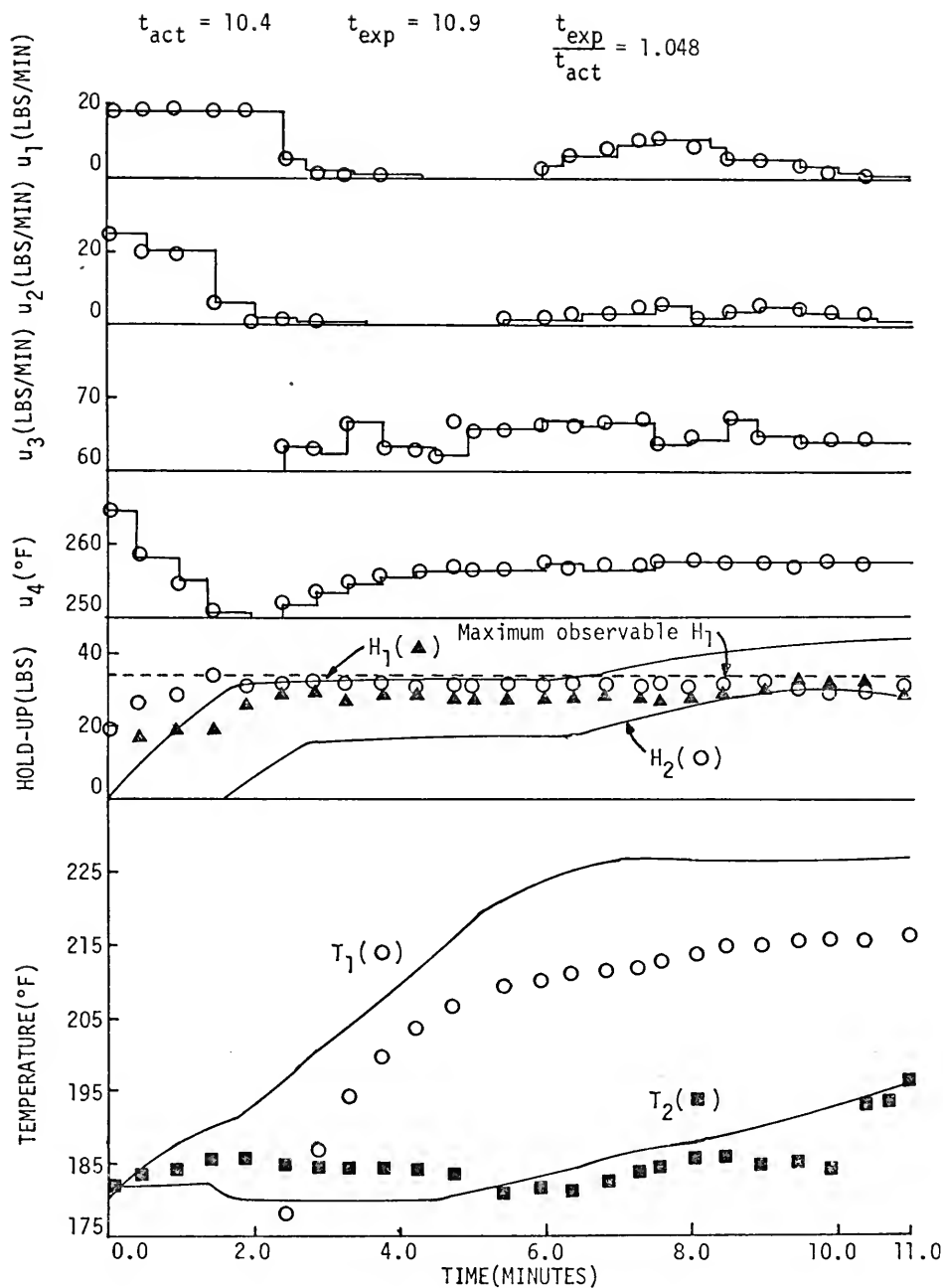


Figure 4.16 Experimental versus Actual Minimum Time for Run C6

TABLE 4.27
EXPERIMENTAL DATA FOR RUN C6

TIME (MINS)	FLOW WF	RATES W12P	IN LBS/MIN W11	W11F	HOLD-UPS(LBS) H1 H2	TEMPERATURES IN DEG F TF T12 T2 TS	COV P
0.06	0.0	5.52	0.0		21.2 27.3	180.6 187.3 178.4 258.1	
0.48	18.06	14.29	0.0		17.0 29.8	180.6 179.8 179.7 258.5	
0.95	14.08	14.78	0.0		26.1 32.2	180.2 176.9 181.9 252.8	
1.25	11.43	13.93	0.0		26.3 33.7	180.2 176.3 182.4 252.8	
1.65	14.64	13.88	0.0		27.6 33.4	180.2 174.8 182.4 254.9	
2.13	10.55	11.24	43.00		30.2 33.7	180.2 174.3 188.5 255.3	
2.60	6.65	7.08	70.09		32.2 33.7	180.4 178.2 189.2 246.1	
3.08	0.0	0.0	70.03		33.4 34.3	180.4 190.3 188.3 252.2	
3.66	0.0	0.0	54.01		32.7 31.4	180.3 201.4 187.3 256.2	
4.02	0.0	0.0	47.98	49.99	31.5 31.4	180.3 203.9 187.3 257.8	0.66
4.49	0.0	0.0	74.10	60.03	31.8 31.1	180.3 204.9 187.3 257.8	0.41
4.96	0.0	0.0	76.63	64.09	31.7 32.2	180.3 207.6 187.0 258.7	0.24
5.48	0.0	0.0	77.38	65.89	31.1 32.2	180.3 210.9 186.5 258.7	0.13
5.97	11.18	0.0	74.76	66.52	30.8 31.6	180.1 213.1 186.3 258.1	0.07
6.38	8.02	0.0	84.44	67.19	30.6 31.9	180.1 213.9 187.6 256.9	0.03
6.85	0.0	0.0	83.95	67.51	31.0 32.9	180.1 214.1 188.9 257.3	0.01
7.32	0.0	0.0	82.03	67.65	31.3 34.0	180.2 214.6 192.9 256.9	0.0
7.79	4.35	3.65	80.47	67.71	31.6 33.7	180.2 214.9 194.6 256.5	0.0
8.32	0.0	0.0	83.01	67.75	30.9 31.6	180.6 215.0 194.6 256.5	0.0
8.73	0.0	4.18	81.45	67.76	31.2 32.2	180.2 214.6 195.1 256.5	0.0
9.02	0.0	0.0	81.55	67.77	31.1 32.5	180.6 214.9 195.1 256.5	0.0
9.49	0.0	6.04	79.67	67.78	31.7 33.7	180.6 215.0 195.5 256.5	0.0
9.91	4.32	0.0	82.60	67.78	31.5 34.0	180.6 215.0 195.5 257.3	0.0
10.16	0.0	0.0	80.73	67.78	31.3 33.7	180.6 215.0 195.5 257.3	0.0
10.67	0.0	0.0	81.09	67.78	31.0 32.5	181.2 215.0 196.1 257.9	0.0
11.09	0.0	0.0	81.19	67.78	32.6 34.9	181.2 217.1 196.5 227.3	0.0

TABLE 4.28

ACTUAL MINIMUM TIME SIMULATION FOR RUN C6

TIME (MINS)	S T A T E		V A R I A B L E S		C O N T R O L			V A R I A B L E S	
	X1 (LBS)	X2 (BTU/LB)	X3 (LBS)	X4 (BTU/LB)	U1 (L B S / M I N)	U2 (L B S / M I N)	U3 (D E G F)	U4 (D E G F)	
0.06	0.0	155.00	25.00	146.39	0.0	5.52	0.0	258.10	
0.56	4.50	163.89	25.44	147.73	12.64	11.66	0.0	258.39	
1.56	17.99	152.99	25.17	148.14	12.10	13.90	0.0	253.33	
2.00	24.06	154.11	25.03	148.00	13.88	13.89	0.0	254.42	
2.31	28.03	155.72	24.93	147.95	11.55	12.03	30.10	255.03	
2.56	31.06	157.11	24.82	147.94	11.55	12.03	30.10	255.03	
2.81	33.38	158.83	24.70	147.93	8.12	8.57	58.09	248.77	
3.06	35.56	160.39	24.59	147.93	8.12	8.57	58.09	248.77	
3.31	36.50	162.30	24.54	147.93	2.44	2.57	66.45	251.20	
3.68	37.47	165.08	24.45	147.92	0.73	0.77	57.74	254.69	
3.93	37.64	166.94	24.48	147.92	0.73	0.77	57.74	254.69	
4.31	37.75	169.56	24.48	147.92	0.22	0.23	50.91	256.87	
4.81	37.79	173.20	24.47	147.92	0.07	0.07	60.03	257.53	
5.31	37.81	176.90	24.47	147.92	0.02	0.02	64.09	258.35	
5.93	37.81	180.69	24.47	149.18	0.01	0.01	65.99	258.59	
6.31	37.80	182.25	26.88	150.26	7.83	0.00	66.52	258.23	
6.68	37.78	183.47	29.90	151.47	7.96	0.00	67.19	257.27	
7.06	37.76	184.61	31.77	152.67	2.39	0.00	67.51	257.27	
7.43	37.74	185.71	32.67	153.95	0.72	0.00	67.65	257.04	
7.81	37.83	186.69	32.89	155.36	3.26	2.55	67.71	256.69	
8.18	38.82	186.96	33.16	156.13	3.26	2.55	67.71	256.69	
8.81	39.70	187.94	33.25	157.91	0.29	3.16	67.76	256.54	
9.06	40.41	188.08	32.58	158.84	0.09	0.95	67.77	256.53	
9.43	40.72	188.81	32.28	160.28	0.09	0.95	67.77	256.53	
10.00	42.63	189.03	30.73	162.03	3.03	1.35	67.78	257.10	
10.37	42.93	189.76	31.06	162.66	0.91	0.41	67.78	257.27	
10.62	43.02	190.30	31.18	163.07	0.91	0.41	67.78	257.27	

LIST OF IMPORTANT VARIABLES USED IN CHAPTER IV

State variables:

- x_1 - First effect hold-up (H_1)
- x_2 - First effect solution enthalpy (h_1)
- x_3 - Second effect hold-up (H_2)
- x_4 - Second effect solution enthalpy (h_2)
- x_5 - First effect solute concentration (C_1)
- x_6 - Second effect solute concentration (C_2)

Control variables:

- u_1 - Feed flow rate to second effect (W_F)
- u_2 - Feed flow (intereffect) rate to first effect (W'_{12})
- u_3 - Recirculation rate in first effect (W_{11})
- u_4 - Steam temperature to first effect (T_S)
- u_5 - Product rate from first effect (W_{01})

Other variables:

- W_{12} - Flow rate of solution entering first effect
- V_{21} - Vapor rate out of first effect
- V_{02} - Vapor rate out of second effect
- h_{12} - Enthalpy of solution entering first effect
- h_1^V - Enthalpy of vapor in first effect
- h_2^V - Enthalpy of vapor in second effect
- C_{12} - Solute concentration in solution entering first effect
- Q_1 - Heat transfer rate in first effect
- Q_2 - Heat transfer rate in second effect

CHAPTER V

COMMENTS AND RECOMMENDATIONS

V.1 Model

The lumped parameter nature of the model causes it to be somewhat inaccurate in predicting the temperatures. There is a temperature gradient in both effects and particularly in the first effect as is evidenced by the temperature gauges at the top and bottom of the tubes. Using a temperature gradient based on these two temperatures is probably better than using a bulk temperature.

The heat transfer mechanisms assumed, particularly the two-phase coefficients and the natural convection coefficient in the second effect, are doubtful. Developed correlations in the literature are of doubtful accuracy. The method for calculating an average heat transfer coefficient in two-phase flow is particularly suspect. All of these suffer some loss of credibility during highly transient conditions. It was hoped that the parameter estimates θ_1 and θ_2 would account for these inaccuracies, but experimental runs C described in Chapter IV indicate that the first effect temperature in particular is not predicted well once boiling has begun.

A better model fit would have been possible by taking θ_1 and θ_2 to be time varying parameters, and these could have been optimally determined in the same fashion as the optimal control policy was determined in Chapter IV. The time varying nature of the parameters

could possibly have accounted for varying heat loss rates throughout the experiment as well as for varying temperature gradients in the effects. All this, of course, would be at the expense of added computer time.

V.2 Experimental Setup

The measurements of the levels in the two effects were subject to a few inaccuracies. Measurement was based on the height of water column in the sight glasses. The actual height varied depending upon the intensity of boiling and the pulsating flow, particularly in the first effect. This drawback could have been alleviated, but certainly not eliminated, had the DP cells which were used on the hold-ups had an adjustable damping device to "filter" the readings.

These momentary variations in the indicated height caused improper analog control of the hold-ups. The proportional gain on the automatic controllers had to be kept high enough to enable the control valves to respond reasonably fast to changes in the hold-ups, but, with a high gain, the momentary variations caused by the boiling in the two effects would cause the controller to respond to the noise. A compromise value of the proportional gain was used, but this did not completely overcome the problem of sensitivity to the momentary variations. A filter on the hold-ups and direct control of the set-points by the IBM 1070 is a possible solution to the problem. This can be done if the vapor rates from the two effects could be directly determined or accurately estimated without a great deal of calculation as this increases the on-line computer costs.

The recirculation flow rate in the first effect was one which did cause a number of problems. Starting the recirculation pump was critical as a certain amount of liquid head on the suction side was necessary to avoid causing the pump to suck in air. Thus the hold-up had to be maintained above a minimum value. In addition, the hold-up had to be kept below a certain value not much above the minimum required to avoid entrainment of the liquid with the vapor going to the second effect. This hold-up was critical and had to be maintained throughout the run.

This problem of a critical hold-up could be avoided if the vapor-liquid separator were baffled or if an additional well-designed separator were installed in addition to the existing one. This would also ensure no entrainment at higher recirculation rates.

Temperature measurement at the exit line of the second effect is not a proper means for obtaining a measure of the temperature in the second effect. This measurement is accurate only if there is a flow out of the second effect. The thermocouple should project directly into the bulk of the second effect liquid in order to get a good estimate of the temperature. The use of additional thermocouples at the top of the two effects may help to give a better temperature average in the two effects and could account for temperature gradients.

The minimum time runs C shown in Chapter IV required the bleeding of the DP cells while the run was in progress. This led to inaccurate flow measurements, but only at the initial time.

V.3 Theory

The theoretical development of the minimum time solution presented in Chapter IV is not a generalized approach as it draws heavily on the concept of a scenario leading to particular orderings of the point constraints. Proposing an optimal scenario is intuitively obvious in some instances. Most start-up problems in chemical engineering processes can be solved by examining relatively few scenarios to determine the optimal one. This approach of ruling out most of the less likely scenarios makes the mathematical problem much less formidable.

In the case of the double effect evaporator, the minimum time solution is bang-bang on the control variables. Although a bang-bang minimum time solution is the rule in linear systems, it is not necessarily true in the case of nonlinear systems such as the evaporator. The switching times found here were directly related to the states thus obtaining the control policy in feedback form. This form made the optimal policy easy to implement in practice.

V.4 Conclusions

In spite of all the drawbacks mentioned in the preceding sections, it can be concluded that the dynamic and algebraic equations of Chapter III together with the parameter estimates for θ_1 and θ_2 provided a reasonably accurate working model of the double effect evaporator. The experimental runs of Section IV.4 show that the model is accurate to within 15 percent in the prediction of the final time. Moreover, with the control policies for minimum time start-up in

feedback form as obtained in Chapter IV, the accuracy of the model does not really matter. The control policy was solely a function of the states and not explicitly a function of time, so that the precise state trajectories predicted by the model were not important from the viewpoint of implementation of the control policy.

The scenario approach adopted in Chapter IV for obtaining the optimal control policy and the resulting use of point constraints not only simplified the mathematics but was a factor in obtaining the control policy in feedback form. This approach could possibly be used with success in other start-up problems in the chemical industry.

APPENDIX A
HEAT TRANSFER EQUATIONS AND OTHER RELATIONSHIPS

In addition to the 6 differential equations (3.1) to (3.6) derived in Chapter III and the 4 algebraic equations (3.7) to (3.9) and (3.11) we have the following algebraic equations to complete the set of equations.

A.1 Relation Between Temperatures and Enthalpies

$$T_1 = f(h_1) \text{ - from solution data} \quad (A.1)$$

$$T_2 = f(h_2) \text{ - from solution data} \quad (A.2)$$

$$h_1^V = f(T_1) \text{ - from steam tables} \quad (A.3)$$

$$h_2^V = f(T_2) \text{ - from steam tables} \quad (A.4)$$

$$T_s = f(h_s) \text{ - from steam tables} \quad (A.5)$$

$$T_{12} = f(h_{12}) \text{ - from solution data} \quad (A.6)$$

$$T_F = f(h_F) \text{ - from solution data} \quad (A.7)$$

A.2 Heat Transfer Equations--First Effect

The mechanism for heat transfer is as presented in (Fair, 1960, 1963a, 1963b), (Hughmark, 1969) and (Tong, 1965). It is assumed that the first effect can be divided into two zones--the sensible heating zone (subscripted s) and the vaporizing zone (subscripted B). Single-phase convective heat transfer is the mechanism in the sensible heating zone whereas the mechanism in the vaporizing zone is a combination of two-phase convective and nucleate boiling heat transfer. Slug flow is

the predominant flow pattern in which the vapor bubbles coalesce into slugs of vapor rapidly once boiling begins. The slugs of gas accelerate more rapidly than the remaining liquid and the vapor has a higher velocity than the liquid at exit. The fractional lengths of the sensible heating zone and the vaporizing zone are obtained through a consideration of the pressure drops relative to the total pressure drop in each tube. The total heat transfer rate is

$$Q_1 = Q_{1s} + Q_{1B} \quad (A.8)$$

A.2.1 Sensible Heating Zone

$$Q_{1s} = U_{1s} A_1 \frac{L_{1s}}{L_1} \Delta T_{1s} \quad (A.9)$$

where the mean temperature difference ΔT_{1s} is

$$\Delta T_{1s} = T_s - (T_1 + T_{12})/2 \quad (A.10)$$

$\left(\frac{L_{1s}}{L_1} \right) A_1$ is the area of the sensible heating zone.

$$Q_{1s} = \hat{h}_{1s}^0 A_1 \frac{L_{1s}}{L_1} (T_s - T_{w2s}) \quad (A.11)$$

and

$$Q_{1s} = \hat{h}_{1s}^i A_1 \frac{L_{1s}}{L_1} (T_{w1s} - T_1) \quad (A.12)$$

where T_{w1s} and T_{w2s} are the inside and outside wall temperatures, respectively. Also

$$Q_{1s} = \hat{h}_{1sw} A_1 \frac{L_{1s}}{L_1} (T_{w1s} - T_{w2s}) \quad (A.13)$$

where \hat{h}_{1sw} is the film coefficient due to the tube walls and fouling.

The outside film coefficient is obtained from the Nusselt equation

$$\hat{h}_{1s}^o = 0.943 \left[\frac{k_{1sf}^3 \rho_{1sf}^2 g \lambda_{1s}}{L_{1s} \mu_{1sf}^2 (T_s - T_{w2s})} \right]^{0.25} \quad (A.14)$$

$$\lambda_{1s} = f(T_s) \quad (A.15)$$

The inside film coefficient is obtained from the Dittus-Boelter equation calculated at boiling zone conditions

$$\hat{h}_{1s}^i = n_1 \frac{k_{1b}}{D_{1i}} \left(\frac{D_{1i} G_1}{\mu_{1b}} \right)^{n_2} \left(\frac{\hat{C}_{1b} \mu_{1b}}{k_{1b}} \right)^{0.4} \quad (A.16)$$

$$\text{For } Re = \left(\frac{D_{1i} G_1}{\mu_{1b}} \right) > 5000; n_1 = 0.023, n_2 = 0.8$$

$$2000 \leq Re \leq 5000; n_1 = 0.0775, n_2 = 0.667$$

$$Re < 2000; n_1 = 0.183, n_2 = 0.545$$

The overall heat transfer coefficient in the sensible heating zone is given by

$$U_{1s} = \frac{1}{\frac{1}{\hat{h}_{1s}^o} + \frac{1}{\theta_1 \hat{h}_{1s}^i} \frac{D_{10}}{D_{1i}} + \frac{1}{\hat{h}_{1sw}}} \quad (A.17)$$

This introduces an unknown parameter θ_1 to account for the uncertainty of the constant used in equation (A.16). $\theta_1 = \theta_{1a}$ when no boiling is taking place and $\theta_1 = \theta_{1b}$ when there is boiling. Also, when there is no boiling $L_{1s} = L_1$.

The length of the sensible heating zone is given by

$$L_{1s} = \frac{L_1 \left(\frac{\Delta T}{\Delta P} \right)_s}{- \left(\frac{\Delta T}{\Delta L} \right) / \left(\frac{\Delta P}{\Delta L} \right) + \left(\frac{\Delta T}{\Delta P} \right)_s} \quad (A.18)$$

where $\left(\frac{\Delta T}{\Delta P} \right)_s$ is the slope of the vapor pressure curve estimated from steam tables

$$\left(\frac{\Delta T}{\Delta P} \right)_s = f(T_1) \quad (\text{A.19})$$

The term $\left(\frac{\Delta T}{\Delta L} \right)$ is obtained from a heat balance

$$\left(\frac{\Delta T}{\Delta L} \right) = \frac{\pi D_{1i} N_{1t} \hat{h}_{1s}^i (T_{w1B} - T_1)}{W_{12} \hat{C}_{1B}} \quad (\text{A.20})$$

The term $\left(\frac{\Delta P}{\Delta L} \right)$ is estimated from

$$\left(\frac{\Delta P}{\Delta L} \right) = - \rho_{1B} g \quad (\text{A.21})$$

The relationship between the properties of liquid occurring in equations (A.14), (A.16), (A.20) and (A.21) at the corresponding temperatures, are

$$T_{1sf} = T_s - 0.75(T_s - T_{w2s}) \rightarrow k_{1sf}, \rho_{1sf}, \mu_{1sf}$$

$$T_{1b} = (T_{12} + T_1)/2 \rightarrow k_{1b}, \mu_{1b}, \hat{C}_{1b}$$

$$T_1 \rightarrow \rho_{1B}, \hat{C}_{1B}$$

A.2.2 Vaporizing Zone

The length is found from the remainder of the total length

$$L_{1B} = L_1 - L_{1s} \quad (\text{A.22})$$

The heat transfer rate is

$$Q_{1B} = U_{1B} A_1 \frac{L_{1B}}{L_1} \Delta T_{1B} \quad (\text{A.23})$$

The mean temperature difference is

$$\Delta T_{1B} = T_s - T_1 \quad (\text{A.24})$$

Also

$$Q_{1B} = \hat{h}_{1B}^o A_1 \frac{L_{1B}}{L_1} (T_s - T_{w2B}) \quad (\text{A.25})$$

$$Q_{1B} = \hat{h}_{1B}^i A_1 \frac{L_{1B}}{L_1} (T_{w1B} - T_1) \quad (\text{A.26})$$

and

$$Q_{1B} = \hat{h}_{1Bw} A_1 \frac{L_{1B}}{L_1} (T_{w2B} - T_{w1B}) \quad (\text{A.27})$$

The outside film coefficient is again obtained from the Nusselt equation

$$\hat{h}_{1B}^o = 0.943 \left[\frac{k_{1Bf}^3 \rho_{1Bf}^2 g \lambda_{1s}}{L_{1B} \mu_{1Bf}^2 (T_s - T_{w2B})} \right]^{0.25} \quad (\text{A.28})$$

The fraction of vapor to liquid is

$$x_{12} = \frac{V_{21}}{W_{12}} \quad (\text{A.29})$$

The variable x'_{12} is introduced for simplifying the solution procedure

$$x'_{12} = 0.4 x_{12} \quad (\text{A.30})$$

The Lockhart-Martinelli parameter X_{tt} , which represents a ratio of kinetic energies of liquid and vapor, is

$$X_{tt} = \left(\frac{1 - x_{12}}{x_{12}} \right)^{0.9} \left(\frac{\rho_{1B}^v}{\rho_{1B}} \right)^{0.5} \left(\frac{\mu_{1B}}{\mu_{1B}^v} \right)^{0.1} \quad (\text{A.31})$$

$$X'_{tt} = \left(\frac{1 - x'_{12}}{x'_{12}} \right)^{0.9} \left(\frac{\rho_{1B}^v}{\rho_{1B}} \right)^{0.5} \left(\frac{\mu_{1B}}{\mu_{1B}^v} \right)^{0.1} \quad (\text{A.32})$$

For the steam-air system the two-phase convective heat transfer coefficient is

$$\hat{h}_{1tp} = 3.5 \hat{h}_{1s}^i \left(\frac{1}{x_{tt}} \right)^{0.5} \quad (\text{A.33})$$

The mass velocity is

$$G_1 = \frac{W_{12}}{N_{1t} S_{1t}} \quad (\text{A.34})$$

The coefficients α used in the calculation of the average inside film coefficient are given by

$$\alpha_{12} = f(G_1, x_{tt}) \quad (\text{A.35})$$

$$\alpha'_{12} = f(G_1, x'_{tt}) \quad (\text{A.36})$$

A mean α is

$$\bar{\alpha}_{12} = (\alpha_{12} + \alpha'_{12})/2 \quad (\text{A.37})$$

The film coefficient due to nucleate boiling is given by

$$\hat{h}_{1nb} = 52(T_{w1B} - T_1) \quad (\text{A.38})$$

The average inside film coefficient is a combination of the nucleate boiling coefficient and the two-phase convective coefficient

$$\hat{h}_{1B}^i = \bar{\alpha}_{12} \hat{h}_{1nb} + \hat{h}_{1tp} \quad (\text{A.39})$$

The overall heat transfer coefficient is

$$U_{1B} = \frac{1}{\frac{1}{\hat{h}_{1B}^o} + \frac{1}{\theta_{1b} \hat{h}_{1B}^i} \frac{D_{10}}{D_{1i}} + \frac{1}{\hat{h}_{1Bw}}} \quad (\text{A.40})$$

where $\frac{1}{\hat{h}_{1Bw}}$ is a combined resistance for the tube walls and fouling.

The temperature-property relationships are

$$T_1 \rightarrow \mu_{1B}, \rho_{1B}^V, \mu_{1B}^V \text{ for liquid}$$

$$T_{1Bf} = T_s - 0.75(T_s - T_{w2B}) \rightarrow k_{1Bf}, \rho_{1Bf}, \mu_{1Bf} \text{ for vapor}$$

A.3 Heat Transfer Equations--Second Effect

$$\lambda_1 = f(T_1) \quad (A.41)$$

$$Q_2 = U_2 A_2 \Delta T_2 \quad (A.42)$$

where the mean temperature difference is given by

$$\Delta T_2 = T_1 - (T_2 + T_F)/2 \quad (A.43)$$

also

$$Q_2 = \hat{h}_2^0 A_2 (T_1 - T_{w2}) \quad (A.44)$$

$$Q_2 = \hat{h}_2^j A_2 (T_{w1} - T_2) \quad (A.45)$$

$$Q_2 = \hat{h}_2^w A_2 (T_{w2} - T_{w1}) \quad (A.46)$$

The outside film coefficient is found from the Nusselt equation

$$\hat{h}_2^0 = 0.943 \left[\frac{k_{2f}^3 \rho_{2f}^2 g \lambda_1}{L_2 \mu_{2f}^2 (T_1 - T_{w2})} \right]^{0.25} \quad (A.47)$$

The inside film coefficient is found from the natural convection equation

$$Gr_2 = \frac{L_2^3 \rho_2^2 g \beta_2 (T_{w1} - T_2)}{\mu_2^2} \quad (A.48)$$

$$Pr_2 = \frac{\hat{C}_2 \mu_2}{k_2} \quad (A.49)$$

$$Y_2 = Gr_2 Pr_2 \quad (A.50)$$

$$\begin{aligned} \hat{h}_2^i &= \left[\frac{0.55 k_2}{L_2} \right] Y_2^{0.25} \text{ for } 10^4 \leq Y_2 < 3.5 \times 10^7 \\ &= \left[\frac{0.13 k_2}{L_2} \right] Y_2^{0.33} \text{ for } Y_2 \geq 3.5 \times 10^7 \end{aligned} \quad (A.51)$$

The overall heat transfer coefficient is

$$U_2 = \frac{1}{\frac{1}{\hat{h}_2^o} + \frac{1}{\theta_2 \hat{h}_2^i} \frac{D_{20}}{D_{2i}} + \frac{1}{\hat{h}_{2w}}} \quad (A.52)$$

The correction factor θ_2 is again introduced to account for uncertainties in the inside film coefficient equation (A.51)

$\theta_2 = \theta_{2a}$ when the liquid is not boiling and $\theta_2 = \theta_{2b}$ when the liquid is boiling. The properties of vapor required in (A.47) are found at the film temperature

$$T_{2f} = T_1 - 0.75(T_1 - T_{w2}) \rightarrow k_{2f}, \rho_{2f}, \mu_{2f}$$

The liquid properties in (A.48), (A.49) and (A.51) are found at the mean temperature

$$\bar{T}_2 = (T_2 + T_{w2})/2 \rightarrow k_2, \rho_2, \mu_2, \hat{C}_2, \beta_2$$

APPENDIX B

LISTING OF COMPUTER PROGRAMS

This appendix contains a listing of the computer programs written for determining the minimum start-up time control policy for the double effect evaporator described in Chapter II. These programs utilize the algorithm of Section IV.2.4. Basically, each iteration consisted of two integrations and for simplicity, two programs were run separately with the output from one being the input to the other.

The first program handled the forward integration of the state equations using either an initial guess on the control policy or an updated version of the previous control policy as input. Integration was done by subroutine HPCG (IBM Scientific Subroutine Package) which is based on a predictor corrector method. Subroutine XDOT supplied the derivatives of the state variables which are the right hand sides of the differential equations (3.1) to (3.6). The results of the forward integration comprising the state and control variables against time were output on cards and on the line printer. The input to the next main program consisted of these cards and the times at which point constraints were encountered on the forward pass together with derivatives of the states before and after these points in time.

The second program integrated the adjoint equations in the reverse direction in time. The Hamiltonian was also minimized at selected points to obtain the optimal control policy. The final conditions and jump conditions on the adjoint variables were evaluated

using the derivatives of the states from the forward integration. Subroutine HPCG was again used for the integration of the adjoint equations and subroutine LAMDOT supplied the right hand sides of the adjoint equations. Subroutine VA04A (Powell, 1964) was the search program used for minimizing the hamiltonian.

Evaluation of the time dependent variables, other than the state and control variables, was done by subroutine FUNCS. The film coefficients were estimated in the following subroutines with the associated references.

- 1) The overall coefficient was evaluated in subroutine HEAT based on the method presented in Fair (1960) and Tong (1965).
- 2) The forced convection coefficient was evaluated in subroutine FCIF using the Dittus-Boelter equation (Hughmark, 1969).
- 3) The natural convection coefficient was evaluated in subroutine FCIN using the natural convection equation (McCabe and Smith, 1967).
- 4) The natural convection coefficient for two phase flow was evaluated in subroutine FCINB using the Rohsenow equation (Tong, 1965).
- 5) The steam side coefficient was evaluated in subroutine FCO using the Nusselt equation (McCabe and Smith, 1967).

```

C *****
C      MAIN PROGRAM FOR INTEGRATION OF THE STATE EQUATIONS
C *****
C      PURPOSE
C      THIS IS THE MAIN PROGRAM WHICH FIRST CALLS SUBROUTINE
C      INPUT TO INPUT DATA. IT THEN INITIALIZES THE STATE
C      VECTOR AND CALLS SUBROUTINE HPCG FROM THE IBM SCIENTIFIC
C      SUBROUTINE PACKAGE TO INTEGRATE THE STATE EQUATIONS.
C      FINALLY, IT OUTPUTS THE RESULTS.
C      SUBPROGRAMS REQUIRED
C      SUBROUTINE INPUT
C      SUBROUTINE HPCG
C
C      DIMENSION X(4),PRMT(5),DERY(4),AUX(16,4)
C      EXTERNAL XDOT,OUT
C      COMMON/TTT/TTT(120)
C      COMMON/STATES/X1(120),X2(120),X3(120),X4(120)
C      COMMON/TIMER/TIME(120),ITIME,IMAX
C      COMMON/STEADY/XHAT(4),XINIT(4),TF,HF,NORDER,U1MAX,U1MIN,
C      U2MAX,U2MIN,U3MAX,U3MIN,U4MAX,U4MIN
C      COMMON/CTRL/U1(120),U2(120),U3(120),U4(120)
C
C      THE INPUT SUBROUTINE IS CALLED HERE. IT INITIALIZES THE
C      COMMON BLOCKS DATA1, DATA2 AND STEADY.
C
C      CALL INPUT
C
C      INITIALIZE ARRAY COUNTER ITIME AND VARIABLES TO BE INPUT
C      TO SUBROUTINE HPCG
C
C      ITIME=1
C      N=4
C      PRMT(1)=0.
C      PRMT(2)=20.
C      PRMT(3)=1.0
C      PRMT(4)=0.2
C      DO 10 J=1,N
10  X(J)=XINIT(J)
C      DERY(1)=3./8.
C      DERY(2)=1./8.
C      DERY(3)=3./8.
C      DERY(4)=1./8.
C      CALL HPCG(PRMT,X,DERY,N,I-FLF,XDOT,OUT,AUX)
C
C      OUTPUT RESULTS ON LINE PRINTER AND ON CARDS
C
C      WRITE(6,2000)
C      IM=ITIME-1
C      WRITE(6,2010)(K,TTT(K),X1(K),X2(K),X3(K),X4(K),K=1,IM)
C      WRITE(6,2020)
C      WRITE(6,2010)(K,TTT(K),U1(K),U2(K),U3(K),U4(K),K=1,IM)

```

```

      WRITE(7,2040) (TTT(K),X1(K),X2(K),X3(K),X4(K),U1(K),U2(K),
1U3(K),U4(K),K=1,IM)
      STOP
2000 FORMAT(1H1,20X,'STATE VARIABLES ON FIRST ITERATION',//,5X,
1'STAGE NO',5X,'TIME',5X,'X1',10X,'X2',10X,'X3',10X,'X4')
2010 FORMAT(7X,I3,7X,F6.3,2X,F9.5,4X,F9.5,4X,F9.5,4X,F9.5)
2020 FORMAT(1H1,20X,'CONTROL VARIABLES ON FIRST ITERATION',//,
15X,'STAGE NO',5X,'TIME',5X,'U1',10X,'U2',10X,'U3',10X,'U4')
2040 FORMAT(F6.2,2X,8F9.4)
      END

```

```

C*****
C                                XDCOT
C*****
C    PURPOSE
C        THIS SUBROUTINE SUPPLIES VALUES OF THE DERIVATIVES TO
C        SUBROUTINE HPCG.
C    USAGE
C        CALL XDCOT(T,X,DERY)
C        NOTEQ WILL BE CALLED ONLY BY SUBROUTINE HPCG
C    PARAMETERS
C        T      = TIME, INPUT
C        X      = STATE VECTOR, INPUT
C        DERY   = VECTOR OF DERIVATIVES OF X AT TIME T, OUTPUT
C    SUBPROGRAMS REQUIRED
C        SUBROUTINE MAP1
C        SUBROUTINE FUNCS
C
C    SUBROUTINE XDCOT(T,X,DERY)
C    DIMENSION X(1),DERY(1),Y(8),IPOST(4),F(8),XX(8),JPOST(4),
C    ID(8)
C    COMMON/INDEX/I(4)
C    COMMON/TIMER/TIME(120),ITIME,IMAX
C    COMMON/TEMPU/UTEMP(4)
C    COMMON/STEADY/XHAT(4),XINIT(4),TF,HF,NORDER,U1MAX,U1MIN,
C    U2MAX,U2MIN,U3MAX,U3MIN,U4MAX,U4MIN
C    COMMON/CTRL/U1(120),U2(120),U3(120),U4(120)
C
C        THE ELEMENTS OF VECTOR I INDICATE WHETHER THE STATES
C        HAVE ARRIVED AT THEIR STEADY STATE VALUES XHAT.
C        EXAMPLE  I(1) = 1  IF X(1).GE.XHAT(1)
C                  = 0  OTHERWISE
C
C    DO 2 N=1,4
C        I(N)=0
C 8  IF(X(N).LT.XHAT(N))GO TO 2
C        I(N)=1
C 2  CONTINUE
C
C        THE VECTOR UTEMP CONTAINS VALUES OF THE CONTROL
C        VARIABLES TO BE USED IN THE PRESENT TIME STEP.  THE
C        OPTIMAL LAW IS UTILIZED HERE IN FEEDBACK FORM.  IF
C        A SIMPLE CONTROL LAW IS NOT AVAILABLE, THEN UTEMP
C        SHOULD BE THE INTERPOLATED VALUES OF THE CONTROL
C        VARIABLES INPUT IN FEEDFORWARD FORM
C
C    UTEMP(1)=U1MAX
C    IF(I(3).EQ.1)UTEMP(1)=U1MIN
C    UTEMP(2)=U2MAX
C    UTEMP(3)=U3MAX
C    IF(I(1).EQ.0)UTEMP(3)=U3MIN
C    UTEMP(4)=U4MAX

```

```

C
C      SUBROUTINE MAP1 IS USED TO MAP THE STATE AND CONTROL
C      VARIABLES INTO A VECTOR D WITH THE STATE VARIABLES
C      IN THE FIRST FOUR POSITIONS IN D AND THE CONTROL
C      VARIABLES IN THE NEXT FOUR
C
40 DO 10 J=1,4
10 IPOST(J)=J
   CALL MAP1(XX,8,X,IPOST,4,Y)
   DC 20 J=5,8
   JJ=J-4
20 JPOST(JJ)=J
   CALL MAP1(Y,8,UTEMP,JPOST,4,D)
C
C      THE VALUES OF THE TIME-DEPENDENT VARIABLES (WHICH ARE
C      FUNCTIONS OF THE STATE AND CONTROL VARIABLES) ARE FOUND
C      BY CALLING SUBROUTINE FUNCS
C
   CALL FUNCS(D,8,F,8)
C
C      RE-MAP THE CONTROL VARIABLES WHICH MAY HAVE BEEN
C      ALTERED BY SUBROUTINE FUNCS
C
DO 25 J=1,4
25 UTEMP(J)=D(J+4)
   W12=F(1)
   V21=F(2)
   V02=F(3)
   H12=F(4)
   Q1=F(5)
   Q2=F(6)
   H1V=F(7)
   H2V=F(8)
C
C      THE RIGHT HAND SIDES OF THE STATE EQUATIONS COMPRISE
C      THE DERIVATIVES
C
DERY(1)=W12-V21-UTEMP(3)
IF(X(1).LT..1E-70)GO TO 50
DERY(2)=(W12*(H12-X(2))+V21*(X(2)-H1V)+Q1)/ X(1)
GO TO 60
50 DERY(2)=0.
GO TO 70
60 DERY(3)=UTEMP(1)-UTEMP(2)-V02
70 IF(X(3).LT..1)GO TO 80
DERY(4)=(UTEMP(1)*(HF-X(4))+Q2+V02*(X(4)-H2V))/ X(3)
IF(I(2).NE.1)DERY(4)=0.
RETURN
80 DERY(4)=0.
RETURN
END

```

```

C *****
C                                     OUT
C *****
C      PURPOSE
C      THIS SUBROUTINE OUTPUTS VALUES OF THE STATE VARIABLES
C      AND DERIVATIVES AT TIME T
C      USAGE
C      CALL OUT(TT,X,DERY,IHLF,NDIM,PRMT)
C      NOTED THIS SUBROUTINE IS CALLED ONLY BY HPCG
C      PARAMETERS
C      ALL ARE INPUTS. REFER TO HPCG FOR DESCRIPTION
C
C      SUBROUTINE OUT(TT,X,DERY,IHLF,NDIM,PRMT)
C      DIMENSION X(1),DERY(1),PRMT(1)
C      COMMON/STATES/X1(120),X2(120),X3(120),X4(120)
C      COMMON/CCNTRL/U1(120),U2(120),U3(120),U4(120)
C      COMMON/TIMER/TIME(120),ITIME,IMAX
C      COMMON/TIME1/TTT(120)
C      COMMON/STEADY/XHAT(4),XINIT(4),TF,HF,NORDER,U1MAX,U1MIN,
1 U2MAX,U2MIN,U3MAX,U3MIN,U4MAX,U4MIN
C      COMMON/TEMPU/UTEMP(4)
C      COMMON/INDEX/I(4)
C      II=ITIME
C
C      THE ERROR PARAMETER PRMT(4) IS RE-ESTIMATED DEPENDING
C      ON THE MAGNITUDE OF X AND THE WEIGHTING
C
C      PRMT(4)=(0.375*ABS(X(1))+0.125*ABS(X(2))+0.375*ABS(X(3))
1 +0.125*ABS(X(4)))*0.01
C
C      A POINT T(I) IS SAVED IN THE OUTPUT ARRAYS (COMMON BLOCK
C      CNTRL,STATES AND TIME1) ONLY IF T(I)-T(I-2) IS GT.0.25
C      MINUTES
C
C      IF(II.LT.3)GO TO 10
C      J=II-1
C      JJ=II-2
C      IF(TT-TTT(JJ)-0.25)5,10,10
5      K=J
C      GO TO 20
10     K=II
C      II=II+1
C      IF(II.LT.121)GO TO 20
C      WRITE(6,2010)II
C      STOP
20     X1(K)=X(1)
C      X2(K)=X(2)
C      X3(K)=X(3)
C      X4(K)=X(4)
C      U1(K)=UTEMP(1)
C      U2(K)=UTEMP(2)

```

```

      U3(K)=UTEMP(3)
      U4(K)=UTEMP(4)
      TTT(K)=TT
      ITIME=II
      IF(K.EQ.J)RETURN
      WRITE(6,2020)TT,(X(J),DERY(J),J=1,NDIM),(PRMT(K),K=1,5)
      IF(I(4).EQ.1)PRMT(5)=1.
      RETURN
2010 FORMAT(5X,'VECTOR OVERFLOW,DIMENSION =',I4)
2020 FORMAT(10X,'OUTPUT FROM      OUT AT TIME=',F10.5,/,5X,
1'STATES',10X,'DERIVATIVES',4(/,3X,F10.5,8X,E15.7),/,1X,'PRM
2T PARAMETERS',5(3X,F10.5))
      END

```



```

C*****
C  MAIN PROGRAM FOR INTEGRATION OF THE ADJOINT EQUATIONS
C*****
C  PURPOSE
C  THIS PROGRAM CALLS THE INPUT ROUTINE TO INITIALIZE
C  COMMON BLOCKS DATA1, DATA2 AND STEADY. IT THEN
C  CALLS ON SUBROUTINE HPCG TO INTEGRATE THE ADJOINT
C  EQUATIONS. WHEN A POINT CONSTRAINT IS ENCOUNTERED ON
C  THE REVERSE INTEGRATION CONTROL RETURNS TO THIS
C  PROGRAM WHICH DETERMINES THE VALUES OF THE ADJOINT
C  VARIABLES
C  SUBPROGRAMS REQUIRED
C  SUBROUTINE SEARCH
C  SUBROUTINE CONVAR
C
C  REAL LAMDA(4)
C  DIMENSION PRMT(5), CLAMDA(4), AUX(16,4), DERY(4)
C  EXTERNAL LAMDCI, LAMOUT
C  COMMON/PCINTS/TF, T2, T1, F3TFM, F4TFM, F2T1P, F2T1M, F3T1P, F3T1M,
1F1T1M
C  COMMON/STATES/X1(120), X2(120), X3(120), X4(120)
C  COMMON/STEADY/XHAT(4), XINIT(4), TF, HF, NORDER, U1MAX, U1MIN,
1U2MAX, U2MIN, U3MAX, U3MIN, U4MAX, U4MIN
C  COMMON/CONTRL/U1(120), U2(120), U3(120), U4(120)
C  COMMON/TIMER/TIME(120), ITIME, IMAX, ICOUNT, TIM, IGO
C  COMMON/CNEW/U1NEW(120), U2NEW(120), U3NEW(120), U4NEW(120)
C  COMMON/DITIME/NITIME
C
C  READ IN VALUES OF THE STATES AND CONTROLS ON THE FORWARD
C  INTEGRATION INTO COMMON BLOCKS STATES, CONTRL AND TIMER.
C  ALSO THE TIMES AT WHICH POINT CONSTRAINTS ARE ENCOUNTERE
D
C  AND THE DERIVATIVES BEFORE AND AFTER THE POINT CONSTRAIN
TS
C
C  CALL INPUT
C  READ(5,1000)IMAX
C  READ(5,1010)(TIME(K), X1(K), X2(K), X3(K), X4(K), U1(K), U2(K),
1U3(K), U4(K), K=1, IMAX)
C  READ(5,1020)F3TFM, F4TFM, T2, T1, F2T1P, F3T1P, F1T1M, F2T1P,
1F2T1M, F3T1M, TF
C  WRITE(6,2030)
C  WRITE(6,2040)(K, TIME(K), X1(K), X2(K), X3(K), X4(K), U1(K),
1U2(K), U3(K), U4(K), K=1, IMAX)
C
C  THE FOLLOWING PROGRAM IS FOR A SCENARIO AS IN PROBLEM 3,
C  CHAPTER 4. GUESS FINAL CONDITION ON LAMDA(3) = 0.008
C  FINAL CONDITIONS FOR THE ADJOINT VARIABLES. NOTE THAT
C  LAMDA(4) IS FOUND FROM THE FINAL CONDITION ON THE
C  HAMILTONIAN
C  IGTC=1

```

```

LAMDA(1)=0.
LAMDA(2)=0.
LAMDA(3)=0.008
LAMDA(4)=(-1.+F3TFM*LAMDA(3))/F4TFM
ISTART=IMAX
IEND=1
PRMT(1)=TIME(IMAX)
PRMT(2)=0.
NITIME=3
PRMT(3)=-0.25
PRMT(4)=0.0025
N=4
ITIME=ISTART
10 CONTINUE
DO 20 J=1,4
20 DLAMDA(J)=0.25
IGO=IGCTC
CALL HPCG(PRMT,LAMDA,DLAMDA,N,IHLF,LAMDOT,LAMOLT,AUX)
IF(IHLF.GT.10)STOP

C
C      OUTPUT RESULTS
WRITE(6,2000)
WRITE(6,2010)(J,TIME(J),X1(J),X2(J),X3(J),X4(J),U1(J),
10 U2(J),U3(J),U4(J),J=ISTART,IEND)
IGCTC=IGCTC+1

C
C      START NEXT INTEGRATION AT TIME WHEN PREVIOUS STAGE ENDED
C
PRMT(1)=TIM
GO TC (70,40,50,70),IGOTO

C
C      CONDITION ON LAMDA(4) AT TIME T2
C
40 CONTINUE
LAMDA(4)=0.
GO TC 10

C
C      CONDITION ON LAMDA(1) AT TIME T1
C
50 CONTINUE
LAMDA(1)=((F2T1P-F2T1M)*LAMDA(2)+(F3T1P-F3T1M)*
1 LAMDA(3))/F1T1M
GO TC 10
70 CONTINUE
STOP
1000 FORMAT(I3)
1010 FORMAT(F6.0,2X,8F9.0)
1020 FORMAT(11F7.0)
2000 FORMAT(1H1,20X,'STATE AND CONTROL VARIABLES',
1 //,2X,'STAGE',2X,'TIME',5X,'X1',10X,'X2',10X,'X3',
210X,'X4',10X,'U1',10X,'U2',10X,'U3',10X,'U4')

```

```

2010 FORMAT(3X,I3,2X,F6.3,2X,F6.3,5X,F7.3,5X,F7.3,5X,F7.3,5X,
1F7.3,5X,F7.3,5X,F7.3,5X,F7.3)
2030 FORMAT(1H1,30X,'STATE AND CONTROL VARIABLES',//,2X,'STAGE',
15X,'TIME',10X,'X1',10X,'X2',10X,'X3',10X,'X4',10X,'U1',10X,
2'U2',10X,'U3',10X,'U4')
2040 FORMAT(4X,I2,5X,F6.2,5X,F7.2,5X,F7.2,5X,F7.2,5X,F7.2,5X,
1F7.2,5X,F7.2,5X,F7.2,5X,F7.2)
END

```

```

C *****
C                                LAMOUT
C *****
C      PURPOSE
C          OUTPUTS VALUES OF LAMDA AND THE DERIVATIVES AT DESIRED
C          TIMES
C      USAGE
C          CALL LAMOUT(TT,LAMDA,DERY,IHLF,NDIM,PRMT)
C              TO BE CALLED ONLY BY SUBROUTINE HPCG
C      PARAMETERS
C          REFER TO SUBROUTINE HPCG FOR PARAMETER DESCRIPTION
C
C      SUBROUTINE LAMOUT(TT,LAMDA,DERY,IHLF,NDIM,PRMT)
C      REAL LAMDA(4),L1(4),L2(4),L3(4),L4(4),L(4)
C      DIMENSION DERY(1),PRMT(1),T(4)
C      COMMON/POINTS/TF,T2,T1,F3TFM,F4TFM,F2T1P,F2T1M,F3T1P,F3T1M,
1F1T1M
C      COMMON/DITIME/NITIME
C      COMMON/TIMER/TIME(120),ITIME,IMAX,ICOUNT,TIM,IGO
C      COMMON/INDEX/I(4)
C      COMMON/CNEW/U1NEW(120),U2NEW(120),U3NEW(120),U4NEW(120)
C
C          CHECK IF PCINT CONSTRAINT IS ENCOUNTERED. IF IT IS
C          ENCOUNTERED TERMINATE INT GRATION AND RETURN TO MAIN
C          PROGRAM FOR NEW VALUES OF ADJOINT VARIABLES
C
C      IGOTO=IGO
C      GO TO (60,70,80),IGOTO
60  CONTINUE
C      IF(TT.LT.T2)PRMT(5)=1.
C      GO TO 80
70  CONTINUE
C      IF(TT.LT.T1)PRMT(5)=1.
80  CONTINUE
C      PRMT(4)=0.0025*(ABS(LAMDA(1))+ABS(LAMDA(2))+ABS(LAMDA(3))+
1ABS(LAMDA(4)))
C      IF(PRMT(4).LT..0009)PRMT(4)=0.0009
C      TIM=TT
C      T(ICCOUNT)=TT
C      L1(ICCOUNT)=LAMDA(1)
C      L2(ICCOUNT)=LAMDA(2)
C      L3(ICCOUNT)=LAMDA(3)
C      L4(ICCOUNT)=LAMDA(4)
C      IF(ICCOUNT.GT.3)GO TO 10
C      ICCOUNT=ICCOUNT+1
C      RETURN
10  TOUT=TIME(ITIME)
C      IF(TOUT.GE.T(4).AND.TOUT.LE.T(1))GO TO 30
C      DO 20 J=1,3
C      T(J)=T(J+1)
C      L1(J)=L1(J+1)

```

```

      L2(J)=L2(J+1)
      L3(J)=L3(J+1)
20    L4(J)=L4(J+1)
      RETURN
30    CALL SEARCH(T,L1,TOUT,4,3,L(1))
      CALL SEARCH(T,L2,TOUT,4,3,L(2))
      CALL SEARCH(T,L3,TOUT,4,3,L(3))
      CALL SEARCH(T,L4,TOUT,4,3,L(4))

C
C      CALL CCNVAR ONLY AT TIMES AT WHICH VARIABLES WERE STORED
C      ON THE FORWARD INTEGRATION
C
      CALL CCNVAR(TOUT,L)
      WRITE(6,2040)TOUT,(L(J),J=1,4)
      WRITE(7,2040)TOUT,(L(J),J=1,4)
2040  FORMAT(F7.3,3X,4F10.4)
2030  FORMAT(I2,F8.3,8F8.3)
      ITIME=ITIME-NITIME
      GO TO 10
      END

```

```

C *****
C                                     LAMDOT
C *****
C      PURPOSE
C      PROVIDES DERIVATIVES OF THE ADJOINT VARIABLES TO
C      SUBROUTINE HPCG
C      USAGE
C      CALL LAMDOT(TT,LAMDA,DERY)
C      NOTED CALLED ONLY BY SUBROUTINE HPCG
C      PARAMETERS
C      TT      - TIME, MINUTES, INPUT
C      LAMDA   - INPUT VECTOR OF ADJOINT VARIABLES
C      DERY    - OUTPUT VECTOR OF DERIVATIVES
C      SUBPROGRAMS REQUIRED
C      SUBROUTINE SEARCH
C      SUBROUTINE FUNCS
C      SUBROUTINE MAP1
C      SUBROUTINE PDERIV
C
C      SUBROUTINE LAMDOT(TT,LAMDA,DERY)
C      REAL LAMDA(4)
C      DIMENSION DERY(1),XX(2),FF(2,2),DFDX(2),W(2),F(8),D(8),Y(2)
C      ,
C      IIPOSIT(2),IMEM(4)
C      COMMON/STATES/X1(120),X2(120),X3(120),X4(120)
C      COMMON/DELT/DELX1,DELX2,DELX3,DELX4,DELU1,DELU2,DELU3,DELU4
C      COMMON/TIMER/TIME(120),ITIME,IMAX,ICOUNT
C      COMMON/INDEX/I(4)
C      COMMON/TEMP/X(8)
C      COMMON/CONTROL/U1(120),U2(120),U3(120),U4(120)
C      COMMON/STEADY/XHAT(4),XINIT(4),TF,HF,NORDER,U1MAX,U1MIN,
C      U2MAX,U2MIN,U3MAX,U3MIN,U4MAX,U4MIN
C      WRITE(6,2060)TT,LAMDA,IMAX
C      2060 FORMAT(10X,'LAMDOT. TT=',F10.3,'LAMDAS=',4(2X,F10.6),2X,'IM
C      AX=',
C      114)
C
C      INTERPOLATE FOR VALUES OF THE CONTROL AND STATE
C      VARIABLES AND SET THOSE CONTROL VARIABLES WHICH CAN
C      BE EXPRESSED IN FEEDBACK FORM
C
C      CALL SEARCH(TIME,X1,TT ,IMAX,NORDER,X(1))
C      CALL SEARCH(TIME,X2,TT ,IMAX,NORDER,X(2))
C      CALL SEARCH(TIME,X3,TT ,IMAX,NORDER,X(3))
C      CALL SEARCH(TIME,X4,TT ,IMAX,NORDER,X(4))
C      CALL SEARCH(TIME,U2,TT ,IMAX,NORDER,X(6))
C      DO 1 J=5,8
C 1 IF(X(J).LT.0.)X(J)=0.
C      IF(X(3).LE.0.)X(3)=0.07
C      IF(X(1).LE.0.)X(1)=0.001
C      DO 2 N=1,4

```

```

      I(N)=0
      IF(X(N).LT.XHAT(N))GO TO 2
      I(N)=1
2  CONTINUE
      X(8)=U4MAX
      X(5)=U1MAX
      IF(I(3).EQ.1)X(5)=U1MIN
      X(7)=U3MAX
      IF(I(1).EQ.0)X(7)=U3MIN
      FEED=X(5)
      CALL FUNCS(X,8,F,8)
      W12=F(1)
      V21=F(2)
      V02=F(3)
      H12=F(4)
      Q1=F(5)
      Q2=F(6)
      H1V=F(7)
      H2V=F(8)
      IGCTC=I(1)+2*I(2)+4*I(3)+8*I(4)+1
      GO TO (10,170,50,170,50,170,50,170,50,50,50,150,50,50,50,12
      5),
1  IGCTC

```

```

      NO STATE VARIABLES AT STEADY STATE
      DETERMINE RIGHT HAND SIDE OF C(LAMDA1)/DT

```

```

10  DERY(4)=0.
13  XX(1)=X(1)
      KCUNT=0
11  CALL PDERIV(XX,FF,2,2,KCUNT,DELX1,DFDX,W)
      IF(KCUNT)5000,16,12
12  Y(1)=XX(KCUNT)
      IPOSIT(1)=1
      CALL MAP1(X,8,Y,IPOSIT,1,D)
      CALL FUNCS(C,8,F,9)
      FF(KCUNT,1)=F(5)
      FF(KCUNT,2)=F(6)
      GO TO 11
16  DQ1DX1=CFEX(1)
      DQ2DX1=CFEX(2)
      IF(V21.LE.0.)DQ2DX1=0.
      DV2DX1=DQ2DX1/(1078.-0.6*X(2))
      DV0DX1=DQ2DX1/(1078.-0.6*X(4))
      DERY(1)=LAMDA(1)*DV2DX1+LAMDA(2)*{W12*(H12-X(2))+V21*(X(2)-
      H1V)+Q1}/(X(1)+X(1))-LAMDA(2)*{(X(2)-H1V)*DV2DX1+DQ1DX1}/X(
      1)+
      2LAMDA(3)*DV0DX1-LAMDA(4)*{DQ2DX1+(X(4)-H2V)*DV0DX1}/X(3)

```

```

      DETERMINE RIGHT HAND SIDE OF C(LAMDA2)/DT

```

```

109 XX(1)=X(2)
    KCUNT=0
106 CALL PDERIV(XX,FF,2,2,KCUNT,DELX2,DFDX,W)
    IF(KCUNT)5000,108,107
107 Y(1)=XX(KCUNT)
    IPCSIT(1)=2
    CALL MAP1(X,8,Y,IPOSIT,1,D)
    CALL FUNCS(D,8,F,8)
    FF(KCUNT,1)=F(5)
    FF(KCUNT,2)=F(6)
    GO TC 106
108 DQ1DX2=DFDX(1)
3000 FORMAT(15X,'LAMBDA(2)=' ,F10.5,5X,'DQ1DX2=' ,E15.7)
    DQ2DX2=DFDX(2)
    IF(V21.LE.0.)DQ2DX2=0.
    DH1DX2=0.
    IF(W12.GT.0.)DH1DX2=X(7)/W12
    DENR=1078.-0.6*D(2)
    DV2DX2=(DENR*DQ2DX2+0.6*Q2)/(DENR*DENR)
    DHVDX2=0.4
    DVODX2=DQ2DX2/(1078.-0.6*X(4))
    IF(VC2.LE.0.)DVODX2=0.
    DERY(2)=-LAMBDA(2)*(W12*(DH1DX2-1.))+DV2DX2*(X(2)-H1V)+
1V21*(1.-CHVDX2)+DQ1DX2)/X(1)-LAMBDA(4)*(DQ2DX2+DVODX2*(X(4)-
2H2V))/X(3)+
    (LAMBDA(1)*DV2DX2+LAMBDA(3)
    *
    3DVODX2)
C
C      DETERMINE RIGHT HAND SIDE OF D(LAMBDA4)/DT
C
155 CONTINUE
    XX(1)=X(4)
    KCUNT=0
160 CALL PDERIV(XX,FF,2,2,KCUNT,DELX4,DFDX,W)
    IF(KCUNT)5000,180,165
165 Y(1)=XX(KCUNT)
    IPOSIT(1)=4
    CALL MAP1(X,8,Y,IPOSIT,1,D)
    CALL FUNCS(D,8,F,8)
    FF(KCUNT,1)=F(5)
    FF(KCUNT,2)=F(6)
    GO TC 160
180 DQ1DX4=DFDX(1)
    DQ2DX4=DFDX(2)
    IF(V21.LE.0.)DQ2DX4=0.
    DH1DX4=0.
    IF(W12.GT.0.)DH1DX4=X(6)/W12
    DV2DX4=DQ2DX4/(1078.-0.6*X(2))
    DENR=1078.-0.6*D(4)
    DVODX4=(DENR*(DQ2DX4-FEED)+0.6*(Q2-FEED*(X(4)-HF)))/(DENR+D
    ENR)

```



```

IF(VC2.LE.0.)DVODX4=0.
DHVDX4=0.4
DERY(4)=-LAMDA(2)*(W12*DH1DX4+(X(2)-H1V)*DV2DX4+DQ1DX4)/X(1
)-
1 LAMDA(4)*(-X(5)+DQ2DX4+(X(4)-H2V)*DVODX4+VO2*(1.-DHVDX4))/X
(3)+
2 (LAMDA(1)*CV2DX4+LAMDA(3)*DVODX4)
IF(I(4).EQ.1)DERY(4)=0.
IF(I(2).EQ.0)DERY(4)=0.
GC TC 112

```

```

C
C      X1, X2 AND X4 AT STEADY STATE
C

```

```

150 CONTINUE
DERY(1)=0.
DERY(4)=0.
GC TC 109

```

```

C
C      X1 AND X2 AT STEADY STATE
C

```

```

170 CONTINUE
DERY(1)=0.
GC TC 109

```

```

C
C      DETERMINE RIGHT HAND SIDE OF D(LAMDA3)/DT
C

```

```

112 XX(1)=X(3)
KOUNT=0
113 CALL PDERIV(XX,FF,2,2,KOUNT,DELX3,DFDX,W)
IF(KOUNT)5000,115,114
114 Y(1)=XX(KOUNT)
IPOSIT(1)=3
CALL MAP1(X,8,Y,IPOSIT,1,C)
CALL FUNCS(C,8,F,8)
FF(KOUNT,1)=F(6)
FF(KOUNT,2)=F(5)
GC TC 113
115 DQ2DX3=DFDX(1)
IF(V21.LE.0.)DQ2DX3=0.
DQ1DX3=DFDX(2)
DV2DX3=DQ2DX3/(1078.-0.6*X(2))
DVODX3=DQ2DX3/(1078.-0.6*X(4))
DERY(3)=-LAMDA(2)*((X(2)-H1V)*DV2DX3+DQ1DX3)/X(1)+LAMDA(3)*
1 DVODX3+LAMDA(4)*(FEED*(HF-X(4))+Q2+VO2*(X(4)-H2V))/(X(3)+X(
3))-
2 LAMDA(4)*(DQ2DX3+(X(4)-H2V)*DVODX3)/X(3)+ (LAMDA(1)*DV2D
X3)
IF(X(3).LT.1.)DERY(3)=0.
GC TC 200

```

```

C
C      ALL VARIABLES AT STEADY STATE

```

C

```

125 DERY(1)=0.
    DERY(2)=0.
    DERY(3)=0.
    DERY(4)=0.
    GO TO 200
5000 WRITE(6,5010)KOUNT
5010 FORMAT(5X,'KOUNT=',I4)
    RETURN
200 CONTINUE
    RETURN

```

C

C

C

```

        ERROR EXIT.  I OUT OF RANGE.  STOP EXECUTION

```

```

50 WRITE(6,2050)(I(J),J=1,4)
  STOP
2050 FORMAT(5X,'LAMDOT. I OUT OF RANGE',4I3)
  END

```

```

C *****
C                                     CONVAR
C *****
C      PURPOSE
C          DETERMINES THE OPTIMAL CONTROL LAW BY MINIMIZATION OF
C          THE HAMILTONIAN
C      USAGE
C          CALL CCNVAR(TOUT,LAMDA)
C      PARAMETERS
C          TOUT   - TIME, MINUTES, AT WHICH CONTROLS ARE TO BE
C                  FOUND, INPUT
C          LAMDA  - INPUT VECTOR CONTAINING ADJOINT VARIABLES AT
C                  TIME TOUT
C      SUBPROGRAMS REQUIRED
C          SUBROUTINE SEARCH
C          SUBROUTINE MAP1
C          SUBROUTINE FUNCS
C          THE FOLLOWING SUBPROGRAMS ARE USED FOR THE MINIMIZATION.
C          THEY ARE NOT LISTED SEPARATELY IN APPENDIX B
C          SUBROUTINE PART1
C          SUBROUTINE PART2
C          SUBROUTINE MAPDOWN
C          SUBROUTINE MAPUP
C          SUBROUTINE VAO4A
C
C      SUBROUTINE CONVAR(TOUT,LAMDA)
C      REAL LAMDA(4)
C      DIMENSION U(4),E(4),V(4),F(8),D(8),X(8),W(60),IPOSIT(4),
1 Z(4),XLO(4),XUP(4),ILO(4),IUP(4),ICLS(4),DX(4)
C      COMMON/DELT/DELT1,DELT2,DELT3,DELT4,DELU1,DELU2,DELU3,DELU4
C      COMMON/CCNTRL/U1(120),U2(120),U3(120),U4(120)
C      COMMON/CATA1/NDATA1,TEMP(22),THALP(22),THALPV(22),
1 ALAMDA(22),PRESS(22),RHOV(22)
C      COMMON/STATES/X1(120),X2(120),X3(120),X4(120)
C      COMMON/STEADY/XHAT(4),XINIT(4),TF,HF,NORDER,U1MAX,U1MIN,
1 U2MAX,U2MIN,U3MAX,U3MIN,U4MAX,U4MIN
C      COMMON/INDEX/I(4)
C      COMMON/CNEW/U1NEW(120),U2NEW(120),U3NEW(120),U4NEW(120)
C      COMMON/TIMER/TIME(120),ITIME,IMAX,ICOUNT
C      DATA ZERC,FRCHG,ESCALE,SC,MAXIT,IP,ICON/1.E-5,0.5,1.E+70,
1 1.E-3,100,1,1/
C
C          INTERPOLATE FOR VALUES OF THE STATE AND ADJOINT
C          VARIABLES AT TIME TOUT
C
C      CALL SEARCH(TIME,X1,TOUT,IMAX,NORDER,X(1))
C      CALL SEARCH(TIME,X2,TOUT,IMAX,NORDER,X(2))
C      CALL SEARCH(TIME,X3,TOUT,IMAX,NORDER,X(3))
C      CALL SEARCH(TIME,X4,TOUT,IMAX,NORDER,X(4))
C      CALL SEARCH(TIME,U2,TOUT,IMAX,NORDER,X(6))
C      TT=TOUT

```

```

IF(X(1).LE.0.)X(1)=0.001
IF(X(3).LE.0.)X(3)=0.07
DO 1 J=5,8
1 IF(X(J).LT.0.)X(J)=0.
DO 3 J=1,4
I(J)=0
IF(X(J).LT.XHAT(J))GO TO 3
I(J)=1
3 CONTINUE

```

C
C
C
C

THE FOLLOWING VALUES OF THE CONTROL VARIABLES WERE
SET BECAUSE OF THE FEEDBACK LAW

```

X(8)=U4MAX
X(5)=U1MAX
IF(I(3).EQ.1)X(5)=U1MIN
X(7)=U3MAX
IF(I(1).EQ.0)X(7)=U3MIN
FEED=X(5)
DO 2 J=1,4
ILG(J)=1
2 IUP(J)=1
IGCTC=I(1)+2*I(2)+4*I(3)+8*I(4)+1
GO TO (500,5,200,5,200,5,200,5,200,200,200,5,200,200,200,
15),IGCTC

```

C
C
C
C
C
C
C
C
C
C

THIS SECTION OF THE SUBPROGRAM IS EXECUTED IN ALL
STAGES EXCEPT WHEN NO STATE VARIABLE IS AT STEADY STATE
T1 T TF

PUT CONTROL VARIABLES THAT HAVE TO BE SEARCHED OVER
IN VECTOR Z

```

5 Z(1)=X(7)
Z(2)=X(8)
Z(3)=X(5)
N=3

```

C
C
C
C
CC

SPECIFY LOWER (XLO) AND UPPER (XUP) BOUNDS ON THE
CONTROL VARIABLES

```

XLO(1)=U3MIN
XLO(2)=U4MIN
XLO(3)=U1MIN
XUP(3)=U1MAX
XUP(1)=U3MAX
XUP(2)=U4MAX

```

C
C

SET UP PARAMETERS FOR MINIMIZATION ROUTINE VAO4A

```

      DC 7 J=1,N
      E(J)=SC*XUP(J)
      RANGE=XUP(J)-XLO(J)
      ESC=RANGE*FRCTG/E(J)
      IF(ESCALE.GT.ESC)ESCALE=ESC
7     CONTINUE
      JTEST=1
10    IT=3

C
C      SUBROUTINE PART1 PARTITIONS THE VARIABLES IN Z INTO
C      THOSE WHICH CANNOT BE SEARCHED OVER AND INTO THOSE WHICH
C      CAN BE SEARCHED OVER. IF A CONSTRAINT IS ENCOUNTERED
C      IT NUMERICALLY EVALUATES THE KUHN-TUCKER MULTIPLIER
C      ASSOCIATED WITH THE PARTICULAR CONTROL VARIABLE TO
C      DETERMINE WHETHER THE CONSTRAINT IS TO BE HELD OR
C      RELEASED. IT ALSO DETERMINES WHEN THE SEARCH IS
C      COMPLETE AFTER ACCOUNTING FOR THE CONSTRAINTS
C
      CALL PART1(Z,E,XLO,XUP,ILC,IUP,ICLS,N,H, JTEST,DX)
      IF(JTEST-4)80,20,90

C
C      SUBROUTINE MAPDOWN MAPS THE UNCONSTRAINED VARIABLES IN Z
C      AS DETERMINED IN PART1 INTO A VECTOR U
C
20    CALL MAPDOWN(Z,E,ICLS ,N,U,V,MU)
      IF(MU.EQ.C)GO TO 10
      GC TO (30,40,50,60),IT

C
C      SUBROUTINE VA04A IS THE CONJUGATE GRADIENT SEARCH
C      PROGRAM WHICH SEARCHES OVER THE VARIABLES IN U TO
C      MINIMIZE THE HAMILTONIAN F
C
30    CALL VA04A1
      GC TO 70
40    CALL VA04A2
      GC TO 70
50    CALL VA04A(U,V,MU,F,ESCALE,IP,ICON,MAXIT,IT,W)
      GC TO 70
60    CALL VA04A4

C
C      SUBROUTINE MAPUP RE-MAPS THE VARIABLES IN VECTOR U INTO
C      VECTOR Z
C
70    CALL MAPUP(Z,ICLS ,N,U,MU)

C
C      SUBROUTINE PART2 CHECKS WHETHER THE SEARCH CAUSED ANY
C      OF THE VARIABLES TO VIOLATE CONSTRAINTS
C
      CALL PART2(Z,XLO,XUP,ILO,IUP,ICLS ,N,IT)
80    CONTINUE

```

```

C      THIS SECTION EVALUATES THE HAMILTONIAN H.
C
      IPOSIT(1)=7
      IPOSIT(2)=8
      IPOSIT(3)=5
      CALL MAP1(X,8,Z,IPOSIT,N,D)
      CALL FUNCS(C,8,F,8)
      DO 87 J=1,8
87  X(J)=C(J)
      W12=F(1)
      V21=F(2)
      V02=F(3)
      H12=F(4)
      Q1=F(5)
      Q2=F(6)
      H1V=F(7)
      H2V=F(8)
      H=LAMDA(2)*(W12*(H12-X(2))+V21*(X(2)-H1V)+Q1)/X(1)+LAMDA(4)
      1*(X(5)*(HF-X(4))+Q2+V02*(X(4)-H2V))/X(3)+LAMDA(3)*(X(5)-
      2X(6)-V02)
      IF(JTEST.EQ.4.AND.IT.NE.5)GO TO 20
      GO TO 10

C      CONVERGENCE IS OBTAINED.  THE OPTIMAL VALUES OF THE
C      CCTRL VARIABLES ARE STORED IN ARRAYS IN COMMON BLOCK
C      CNEW
C
      90 U3NEW(ITIME)=Z(1)
      U4NEW(ITIME)=Z(2)
      U1NEW(ITIME)=Z(3)
      U2NEW(ITIME)=X(6)
      RETURN

C      THIS SECTION OF THE SUBPROGRAM IS EXECUTED WHEN NO
C      STATE VARIABLES ARE AT STEADY STATE.  T T1
C
      500 Z(1)=X(6)
      Z(2)=X(8)
      N=2
      XLO(1)=U2MIN
      XLO(2)=U4MIN
      XUP(1)=U2MAX
      XUP(2)=U4MAX
      DO 505 J=1,N
      E(J)=SC*XUP(J)
      RANGE=XUP(J)-XLO(J)
      ESC=RANGE*FRCHG/E(J)
      IF(ESCALE.GT.ESC)ESCALE=ESC
      505 CONTINUE
      JTEST=1
      510 IT=3

```

```

      CALL PART1(Z,E,XLO,XUP,ILO,IUP,ICLS,N,H, JTEST,DX)
511  CONTINUE
      IF(JTEST-4)580,520,600
520  CALL MAPDOWN(Z,E,ICLS ,N,U,V,MU)
      IF(MU.EQ.0)GO TO 510
      GO TO (530,540,550,560),IT
530  CALL VAQ4A1
      GO TO 570
540  CALL VAQ4A2
      GO TO 570
550  CALL VAQ4A(U,V,MU,H,ESCALE,IP,ICON,MAXIT,IT,W)
      GO TO 570
560  CALL VAQ4A4
570  CALL MAPUP(Z,ICLS ,N,U,MU)
      CALL PART2(Z,XLO,XUP,ILO,IUP,ICLS ,N,IT)
571  CCNTINUE
580  IPCSIT(1)=6
      IPOSIT(2)=8
      CALL MAP1(X,8,Z,IPCSIT,N,C)
      CALL FUNCS(C,8,F,8)
      DO 590 J=1,8
590  X(J)=C(J)
591  CONTINUE
      W12=F(1)
      V21=F(2)
      VC2=F(3)
      H12=F(4)
      Q1=F(5)
      Q2=F(6)
      H1V=F(7)
      H2V=F(8)
      H=LAMDA(1)*(W12-V21-X(7))      +LAMDA(2)*(W12*(H12-X(2))+V21
                                      *(X(2)
1-H1V)+Q1)/X(1)+LAMDA(3)*(X(5)-X(6)-VC2)+LAMDA(4)*(X(5)*(HF-
                                      X(4)))
      2+Q2+VC2*(X(4)-H2V))/X(3)
592  CCNTINUE
      IF(JTEST.EQ.4.AND.IT.NE.5) GO TO 520
      GO TO 510
600  U2NEW(ITIME)=Z(1)
      U3NEW(ITIME)=C.
      U4NEW(ITIME)=Z(2)
      U1NEW(ITIME)=X(5)
      RETURN
C
C      ERROR EXIT.  I OUT OF RANGE.  STOP EXECUTION
C
200  WRITE(6,2050)((I(J),J=1,4)
2050  FORMAT(2X,'IN CONVAR.  I=',4(2X,I2))
      STOP
      END

```

```

C *****
C                                     INPUT
C *****
C      PURPCSE
C          TC INPUT DATA FOR COMMON BLOCKS DATA1, DATA2, STEADY
C          AND DELT
C      USAGE
C          CALL INPUT
C      SUBPRCGRAMS REQUIRED
C          SUBROUTINE SEARCH
C
C      SUBROUTINE INPUT
C          COMMON/DATA1/NCATA1,TEMP(22),THALP(22),THALPV(22),
1ALAMDA(22),PRESS(22),RHOV(22)
C          COMMON/STEADY/XHAT(4),XINIT(4),TF,HF,NORDER,U1MAX,U1MIN,
1U2MAX,U2MIN,U3MAX,U3MIN,U4MAX,U4MIN
C          COMMON/DELT/DELX1,DELX2,DELX3,DELX4,DELU1,DELU2,DELU3,DELU4
C          COMMON/DATA2/NCATA2,T(8),RHO(8),CP(8),VISC(8),THCOND(8),
1PRANTL(8),BETA
C
C          READ NUMBER OF POINTS IN ARRAYS IN DATA1 AND DATA2,
C          CRDER OF INTERPOLATING POLYNOMIAL, FEED TEMPERATURE
C          AND EXPANSION COEFFICIENT OF WATER
C
C      READ(5,1000)NCATA1,NCATA2,NORDER,TF,BETA
C
C          READ PROPERTIES OF WATER AND STEAM VS TEMPERATURE, I.E.
C          ENTHALPY,VAPOR ENTHALPY, LATENT HEAT, SATURATION
C          PRESSURE AND VAPOR DENSITY
C
C      READ(5,1010)(TEMP(I),THALP(I),THALPV(I),ALAMDA(I),PRESS(I),
1RHOV(I),I=1,NCATA1)
C
C          READ PROPERTIES OF WATER VS TEMPERATURE, I.E. DENSITY,
C          SPECIFIC HEAT, VISCOSITY, THERMAL CONDUCTIVITY AND
C          PRANDTL NUMBER
C
C      READ(5,1010)(T(I),RHO(I),CP(I),VISC(I),THCOND(I),PRANTL(I),
1I=1,NCATA2)
C
C          FIND FEED ENTHALPY CORRESPONDING TO FEED TEMPERATURE
C
C      CALL SEARCH(TEMP,THALP,TF,NCATA1,NORDER,HF)
C
C          READ STEADY STATE VALUES
C
C      READ(5,1030)XHAT(1),T1SS,XHAT(3),T2SS,X3MAX
C
C          DETERMINE STEADY STATE ENTHALPIES AND TEMPERATURES
C

```



```
CALL SEARCH(TEMP,THALP,T1SS,NCDATA1,NORDER,XHAT(2))
CALL SEARCH(TEMP,THALP,T2SS,NCDATA1,NORDER,XHAT(4))
```

```
      READ INITIAL VALUES OF STATE VARIABLES
```

```
      READ(5,1030)(XINIT(I),I=1,4)
```

```
      READ MAXIMUM AND MINIMUM VALUES OF CONTROL VARIABLES
```

```
      READ(5,1040)U1MAX,U1MIN,U2MAX,U2MIN,U3MAX,U3MIN,U4MAX,U4MIN
```

```
      READ INITIAL PERTURBATIONS IN STATE AND CONTROLS FOR
      USE IN SUBROUTINE PDERIV
```

```
      READ(5,1040)DELX1,DELX2,DELX3,DELX4,DELU1,DELU2,DELU3,DELU4
```

```
      OUTPUT DATA READ IN
```

```
      WRITE(6,2000)
```

```
      WRITE(6,2010)(TEMP(I),THALP(I),THALPV(I),ALAMDA(I),
```

```
1P PRESS(I),RHOV(I),I=1,NCDATA1)
```

```
      WRITE(6,2020)
```

```
      WRITE(6,2030)(T(I),RHO(I),CP(I),THCOND(I),VISC(I),
```

```
1PRANDTL(I),I=1,NCDATA2)
```

```
      WRITE(6,2040)TF,HF,BETA
```

```
      WRITE(6,2050)(I,XINIT(I),XHAT(I),I=1,4)
```

```
      WRITE(6,2080)
```

```
1WRITE(6,2090)U1MAX,U1MIN,U2MAX,U2MIN,U3MAX,U3MIN,U4MAX,
U4MIN
```

```
      WRITE(6,3040)DELX1,DELX2,DELX3,DELX4,DELU1,DELU2,DELU3,
```

```
1DELU4
```

```
      RETURN
```

```
1000 FORMAT(3I2,3F10.0)
```

```
1010 FORMAT(6F10.0)
```

```
1020 FORMAT(I2)
```

```
1030 FORMAT(5F10.0)
```

```
1040 FORMAT(8F10.0)
```

```
1060 FORMAT(1X,I2,2X,F6.3,F10.7,F10.6,F10.7,F10.6,3F8.5,F5.1)
```

```
2000 FORMAT(1H1,10X,'PROPERTIES OF WATER AND STEAM',//,5X,'TEMP'
1,2X,'LIQ ENTHALPY',2X,'VAP ENTHALPY',2X,'LATENT HEAT',3X,
2'PRESSURE',2X,'VAP DENSITY',/,4X,'(DEG F)',3X,'(BTU/LB)',5X
3,'(BTU/LB)',7X,'(BTU/LB)',5X,'(PSI)',4X,'(LBS/CU FT)')
```

```
2010 FORMAT(4X,F5.1,4X,F6.2,7X,F7.2,8X,F7.2,5X,F7.4,4X,F7.5)
```

```
2020 FORMAT(//5X,'TEMP',3X,'DENSITY',4X,'SP.HT.',2X,'TH.COND.',
13X,'VISCOSITY',2X,'PRANDTL NO',/,4X,'(DEG F)',1X,
2'(LBS/CUFT)',1X,'(B/LB.F)',1X,'(BF/HSQFF)',1X,'(LBS/FTHR)')
```

```
2030 FORMAT(4X,F5.1,4X,F5.2,5X,F6.4,5X,F5.3,4X,F6.4,5X,F5.3)
```

```
2040 FORMAT(1H1,5X,'STEADY STATE AND INITIAL VALUES',/,5X,
1'FEED TEMPERATURE',F5.1,/,5X,'FEED ENTHALPY' =',F7.2,/,5X,
2'BETA',12X,'=',F9.7,//8X,'XINIT',6X,'XHAT')
```

```
2050 FORMAT(5X,I1,2X,F5.2,5X,F6.2)
```

```

2090 FORMAT(1H1,5X,'MAX AND MIN VALUES OF CONTROL VARIABLES',//,
117X,'MAX VALUE',5X,'MIN VALUE')
2090 FORMAT(18X,F7.3,7X,F7.3)
3040 FORMAT(1H1,10X,'INITIAL INCREMENT IN INDEPENDENT VARIABLES'
1,',//,5X,'DELX1=',F5.2,/,5X,'DELX2=',F5.2,/,5X,'DELX3=',F5.2,
2/,5X,'DELX4=',F5.2,/,5X,'DELU1=',F5.2,/,5X,'DELU2=',F5.2,/,
35X,'DELU3=',F5.2,/,5X,'DELU4=',F5.2)
END

```

```

C *****
C                                     SEARCH
C *****
C   PURPOSE
C       TO INTERPOLATE USING A POLYNOMIAL OF DESIRED ORDER
C   USAGE
C       CALL SEARCH(X,Y,XGIVEN,NCDATA,NORDER,YFOUND)
C   DESCRIPTION OF PARAMETERS
C       X       -INPUT VECTOR OF INDEPENDENT VARIABLE
C       Y       -INPUT VECTOR OF DEPENDENT VARIABLE
C       XGIVEN-INPUT POINT AT WHICH VALUE OF Y IS REQUIRED
C       NCDATA -INPUT NUMBER OF DATA POINTS
C       NORDER-INPUT ORDER OF INTERPOLATING POLYNOMIAL
C       YFOUND-OUTPUT VALUE OF DEPENDENT VARIABLE
C   REMARKS
C       MAX DIMENSION OF INPUT VECTOR IS 20 AND MAX ORDER OF
C       INTERPOLATING POLYNOMIAL IS 4 (IF GREATER,CHANGE
C       DIMENSION STATEMENTS ACCORDINGLY)
C   SUBPROGRAMS REQUIRED
C       FUNCTION AITKEN(X,Y,NCDATA,XGIVEN)
C   METHOD
C       TAKES NORDER+1 POINTS AND INTERPOLATES USING AITKEN-
C       LAGRANGE INTERPOLATION
C
C   SUBROUTINE SEARCH(X,Y,XGIVEN,NCDATA,NORDER,YFOUND)
C   DIMENSION X(20),Y(20),A(5),B(5)
C
C   SEARCH FOR NORDER POINTS
C
C   YFOUND=0.
C   DO 30 M=1,NCDATA
C     IF(XGIVEN-X(M))10,20,30
10  JJ=M
C   GO TO 40
20  YFOUND=Y(M)
C   RETURN
30  CONTINUE
40  LL=JJ-NORDER/2+1
C   IF(LL.LE.0)LL=1
C   LU=LL+NORDER-1
C   IF(LU.LT.NCDATA) GO TO 50
C   LU=NCDATA
C   LL=LU-NORDER+1
50  DO 60 M=LL,LU
C     K=M-LL+1
C     A(K)=X(M)
C     B(K)=Y(M)
60  CONTINUE
C   IF(YFOUND.EQ.0)YFOUND=AITKEN(A,B,NORDER,XGIVEN)
C   RETURN

```

END

```

C *****
C                                     AITKEN
C *****
C  PURPOSE
C  INTERPOLATES USING POLYNOMIAL OF DESIRED ORDER
C  USAGE
C  FUNCTION AITKEN(X,Y,NDATA,XGIVEN)
C  DESCRIPTION OF PARAMETERS
C  X      -INPUT VECTOR OF INDEPENDENT VARIABLE
C  Y      -INPUT VECTOR OF DEPENDENT VARIABLE
C  NDATA  -NUMBER OF DATA POINTS
C  XGIVEN -INPUT POINT AT WHICH VALUE OF Y IS REQUIRED
C  REMARKS
C  XGIVEN MUST LIE IN THE RANGE OF THE TABLE. IF NDATA IS
C  GREATER THAN 5, ADJUST DIMENSION STATEMENT
C  METHOD
C  AITKEN-LAGRANGE INTERPOLATION
C
C  FUNCTION AITKEN(X,Y,NDATA,XGIVEN)
C  DIMENSION X(5),Y(5),Z(5,5)
C  DO 10 I=1,NDATA
10  Z(I,1)=Y(I)
C  NN=NDATA-1
C  DO 20 L=1,NN
C  LL=L+1
C  DO 30 K=LL,NDATA
30  Z(K,L+1)=((X(K)-XGIVEN)*Z(L,L)-(X(L)-XGIVEN)*Z(K,L))/(X(K)-
C                                     X(L))
C  20 CONTINUE
C  AITKEN=Z(NDATA,NDATA)
C  RETURN
C  END

```

```

C*****
C                                     FUNCS
C*****
C      PURPOSE
C          CALCULATES VALUES OF ALGEBRAIC VARIABLES FROM
C          VALUES OF CONTROL AND DIFFERENTIATED VARIABLES.
C
C      USAGE
C          CALL FUNCS(X,N,F,M)
C
C      DESCRIPTION OF PARAMETERSO
C          X - N DIMENSIONAL VECTOR OF DIFFERENTIATED AND
C          CONTROL VARIABLES AS DESCRIBED IN SUBROUTINE XDOT
C          F - M DIMENSIONAL VECTOR CONTAINING VALUES OF
C              ALGEBRAIC VARIABLES,WHERE,
C              F(1) - FEED TO FIRST EFFECT, W12 (LBS/MIN)
C              F(2) - VAPOR FROM FIRST EFFECT, V21 (LBS/MIN)
C              F(3) - VAPOR FROM SECOND EFFECT, V02 (LBS/MIN)
C              F(4) - ENTHALPY OF FEED TO FIRST EFFECT, H12,
C                      BTU/LB
C              F(5) - HEAT TRANSFER IN FIRST EFFECT, Q1,
C                      BTU/MIN
C              F(6) - HEAT TRANSFER IN SECOND EFFECT, Q2
C                      BTU/MIN
C              F(7) - VAPOR ENTHALPY IN FIRST EFFECT, H1V,
C                      BTU/LB
C              F(8) - VAPOR ENTHALPY IN SECOND EFFECT, H2V,
C                      BTU/LB
C
C      SUBPROGRAMS REQUIRED
C          SUBROUTINE HEAT
C          SUBROUTINE SEARCH
C
C          IN THE RANGE T=212 DEG F TO 220 DEG F (I.E. ENTHALPY
C          180.07 BTU/LB TO 188.13 BTU/LB)
C      RELATIONSHIP BETWEEN VAPOR ENTHALPY AND TEMPERATUREO
C          HV = 1066.0 + 0.4*T
C      RELATIONSHIP BETWEEN TEMPERATURE AND LIQUID ENTHALPYO
C          T = HL + 32.0
C      WHERE,
C          T = TEMPERATURE (DEG F)
C          HL = LIQUID ENTHALPY (BTU/LB)
C          WHERE HV = VAPOR ENTHALPY (BTU/LB)
C          RHO = VAPOR DENSITY (LBS/CU.FT)
C
C      DIMENSION X(1),F(1)
C      COMMON/CAT1/NDATA1,TEMP(22),THALP(22),THALPV(22),
C      1ALAMDA(22),PRESS(22),RHOV(22)
C      COMMON/STEADY/XHAT(4),XINIT(4),TF,HF,NORDER,U1MAX,U1MIN,
C      1U2MAX,U2MIN,U3MAX,U3MIN,U4MAX,U4MIN
C      COMMON/INDEX/I(4)
C
C      FIND TEMPERATURES AND VAPOR ENTHALPIES IN EFFECTS
C

```

```

5  T1=X(2)+32.0
   T2=X(4)+32.0
   H1V=1066.+0.4*T1
   H2V=1066.+0.4*T2
   IF(I(2).EQ.1)GO TO 10

```

C
C
C

```

      X2.NE.XHAT(2), FIRST EFFECT IS BEING HEATED.

```

```

      Q2=0.
      V21=0.
      VC2=0.
      W12=X(6)+X(7)
      IF(W12.NE.0.)GO TO 1
      T12=T1
      GC TC 2
1  H12=(X(7)*X(2)+X(6)*X(4))/W12
   T12=H12+32.
2  CALL HEAT(Q1,X(1),W12,V21,X(8),T1,T12,1,1)
   GC TC 70

```

C
C
C

```

      X2.EQ.XHAT(2), FIRST EFFECT IS BOILING.

```

```

10 CONTINUE
   CALL HEAT(Q2,X(3),0.,0.,T1,T2,TF,3,2)
   CALL SEARCH(TEMP,ALAMDA,T1,NDATA1,NORDER,ALAM1)
   V21=Q2/ALAM1
   IF(I(1).EQ.1)X(6)=V21
   W12=X(6)+X(7)
   IF(W12.NE.C.)GO TO 190
   T12=T1
   GC TC 200
190 T12=(X(7)*T1+X(6)*T2)/W12
   H12=T12-32.
200 CALL HEAT(Q1,X(1),W12,V21,X(8),T1,T12,2,1)
   IF(I(4).EQ.0)GO TO 70

```

C
C
C

```

      X4=XHAT(4), SECOND EFFECT IS ALSO BOILING.

```

```

      CALL SEARCH(TEMP,ALAMDA,T2,NDATA1,NORDER,ALAM2)
      VQ2=(Q2-X(5)*(X(4)-HF))/ALAM2
      GC TC 80
70 VC2=0.
90 F(1)=W12
   F(2)=V21
   F(3)=VQ2
   F(4)=H12
   F(5)=Q1
   F(6)=Q2
   F(7)=H1V
   F(8)=H2V
   RETURN

```

END


```

C *****
C                                     HEAT
C *****
C  PURPOSE
C    EVALUATES THE HEAT TRANSFER RATE
C  USAGE
C    CALL HEAT(Q,H,W,V,TO,TI,TIN,IPHASE,IEFFECT)
C  PARAMETERS
C    Q      - HEAT TRANSFER RATE, BTU/MIN, OUTPUT
C    H      - HOLD-UP, LBS, INPUT
C    W      - LIQUID FLOW RATE, LBS/HR, INPUT
C    V      - VAPOR FLOW RATE, LBS/HR, INPUT
C    TO     - OUTSIDE (STEAM SIDE) TEMPERATURE, DEG F, INPUT
C    TI     - INSIDE (LIQUID SIDE) TEMPERATURE, DEG F, INPUT
C    TIN    - INLET TEMPERATURE OF LIQUID, DEG F, INPUT
C    IPHASE - PARAMETER DESCRIPTIVE OF TYPE OF BOILING
C              = 1, INDICATES SINGLE PHASE FLOW
C              = 2, INDICATES TWO PHASE FLOW
C              = 3, INDICATES NATURAL CONVECTION
C    IEFFECT - PARAMETER INDICATING EFFECT FOR WHICH HEAT
C              TRANSFER RATE IS TO BE CALCULATED, INPUT
C  SUBPRGGRAMS REQUIRED
C    FUNCTION FCINB
C    FUNCTION FCIN
C    FUNCTION FCO
C    FUNCTION FCIF
C    FUNCTION HEIGHT
C    SUBROUTINE ALPHA
C
C  SUBROUTINE HEAT(Q,H,W,V,TO,TI,TIN,IPHASE,IEFFECT)
C  REAL L1S,L1,L1B
C  DIMENSION THETA(3)
C  COMMON/DATA1/NDATA1,TEMP(22),THALP(22),THALPV(22),
C  1ALAMDA(22),PRESS(22),RHOV(22)
C  COMMON/DATA2/NDATA2,T(8),RHO(8),CP(8),VISC(8),THCOND(8),
C  1PRANTL(8),BETA
C  COMMON/INDEX/I(4)
C  DATA NORDER/4/
C  IF(TC.LE.TI)GO TO 25
C
C    L1 IS THE MAXIMUM LENGTH OF THE TUBES IN FEET.
C    A IS THE MAXIMUM HEAT TRANSFER AREA IN SQ.FT.
C
C  L1=9.5
C  IF(IEFFECT.EQ.2)L1=1.917
C  A=7.46128
C  IF(W.EQ.0.)TIN=TI
C  TBULK=(TIN+TI)*0.5
C  IF(V.LE.0.)IPHASE=1
C  IF(IEFFECT.EQ.2)A=9.08116

```

THE OUTER WALL TEMPERATURE, TW2 AND INNER WALL
TEMPERATURE TW1 ARE GUESSED INITIALLY AND THEN ITERATED
ON BY EVALUATION OF THE FILM COEFFICIENTS AND THEN
RE-EVALUATING THE WALL TEMPERATURES

TW2=0.5*(TC+TBULK)
TW1=TW2
ITIME=ITIME+1

THE PARAMETERS THETA ARE CORRECTION PARAMETERS FOR THE
TWC EFFECTS

THETA1 = 0.1395 WHEN FIRST EFFECT IS NOT BOILING
= 0.0928 OTHERWISE
THETA2 = 0.1359 WHEN SECOND EFFECT IS NOT BOILING
= 0.1763 OTHERWISE

THETA(1)=0.1395
THETA(2)=1.
THETA(3)=1.
IF(I(2).EQ.0)GO TO 5
THETA(1)=0.0928
THETA(2)=0.1359
THETA(3)=0.1763

5 CONTINUE

KCOUNT=1

TH1=THETA(1)

10 IF(IEFFECT.EQ.1)GO TO 11

TH1=THETA(2)

IF(I(4).EQ.1)TH1=THETA(3)

GO TO 12

11 IF(W.NE.0.)GO TO 15

12 IF(I(4).NE.1.OR.IEFFECT.NE.2)GO TO 13

HIS IS THE INSIDE FILM COEFFICIENT
HOS IS THE OUTSIDE FILM COEFFICIENT
TW1N AND TW2N ARE THE NEW INNER AND OUTER WALL
TEMPERATURES

HIS=FCIN(TW1,TH1)

GO TO 17

13 HIS=FCIN(TBULK,TW1,H,IEFFECT)

GO TO 17

15 HIS=FCIF(TBULK,W,TW1)

17 HOS=FCO(TC,TW2,IEFFECT)

HW=849.624

TW1N=(TBULK*TH1 *HIS*(HOS+HW)+HW*HOS*TO)/(TH1 *HIS*
1 (HOS+HW)+HOS*HW)

TW2N=(HOS*TO+HW*TW1N)/(HOS+HW)

IF(ABS(TW1N-TW1).LT.1..AND.ABS(TW2N-TW2).LT.1.)GO TO 20

IF(KCOUNT.GT.15)WRITE(6,2100)HIS,HOS,TW1,TW2,TW1N,TW2N

IF(I(4).NE.1.OR.IEFFECT.NE.2.OR.KCOUNT.GT.10)GO TO 18

```

      TW1=TW1+0.2*(TW1N-TW1)
      TW2=TW2+0.2*(TW2N-TW2)
      GC TC 19
18  TW1=TW1N
      TW2=TW2N
19  KCOUNT=KCOUNT+1
      IF(KCOUNT.LT.20)GO TO 10
      WRITE(6,2000)KCOUNT,TC,TI,TIN,IPHASE,IEFFECT
      STOP
20  IF(ABS(HIS).LT..1E-70)GO TO 25

C
C      CALCULATE OVERALL HEAT TRANSFER COEFFICIENT UIS IN
C      SENSIBLE HEATING ZONE
C      LIS IS THE LENGTH OF THE SENSIBLE HEATING ZONE
C      DELT1S IS THE MEAN TEMPERATURE DIFFERENCE IN THE
C      SENSIBLE HEATING ZONE
C      Q1S IS THE HEAT TRANSFER RATE IN THE SENSIBLE HEATING
C      ZONE
C
      UIS=(1./((1.25357/(TH1      *HIS)+1./HOS+0.001315+0.002)))/60.
      LIS=HEIGHT(IPHASE,H,TBULK,TW1,TIN,W,IEFFECT)
      DELT1S=TC-(TI+TIN)*0.5
      Q1S=UIS*A *LIS*DELT1S/L1
      IF(IPHASE.EQ.2)GO TO 30
      Q=Q1S
      RETURN
25  Q=0.
      RETURN

C
C      THIS PART OF THE SUBROUTINE IS USED ONLY FOR THE
C      CALCULATION OF THE HEAT TRANSFER RATE IN THE FIRST
C      EFFECT WHEN BOILING OCCURS. REFER TO APPENDIX A FOR
C      THE EQUATIONS
C
30  TW2=0.5*(TC+TI)
      TW1=TW2
      ITIME=ITIME+1
31  CONTINUE
      TB=TI
      KCOUNT=1
32  IF((W-V).GT.0.)GO TO 35
      X12=1.
      H1B=22.15727*V**0.8
      GC TC 40
35  X12=V/W
      H1B=FCIF(TB,w,TW1)
40  HCB=FCO(TC,TW2,IEFFECT)
      HW=849.624
      TW1N=(TB*THETA(1)*H1B*(HOB+HW)+HW*HOB*TO)/(THE TA(1)*
1  H1B*(HOB+HW)+HOB*TW)
      TW2N=(HCB*TC+FW*TW1N)/(HOB+HW)

```

```

IF(ABS(TW1N-TW1).LT.1..AND.ABS(TW2N-TW2).LT.1. )GO TO 50
TW1=TW1N
TW2=TW2N
IF(TW1.LT.TI)TW1=TI
IF(TW2.GT.TO)TW2=TO
KCUNT=KCUNT+1
IF(KCUNT.LT.10)GO TO 32
WRITE(6,2000)KOUNT,TC,TI,TIN,IPHASE,IEFECT
STOP
50 IF((W-V).LE.0.)GO TO 65
X12PR=0.4*X12
CALL SEARCH(T,RHO,TB ,NDATA2,NORDER,RHOL)
CALL SEARCH(T,VISC,TB ,NDATA2,NORDER,VISCL)
CALL SEARCH(TEMP,RHOV,TI,NDATA1,NORDER,RHOG)
VISC=0.0195329+5.107E-05*TI
XTT=((1.-X12)/X12)**0.9*(RHOG/RHOL)**0.5*(VISCL/VISC)**0.1
XTTPR=((1.-X12PR)/X12PR)**0.9*(RHOG/RHOL)**0.5*(VISCL/VISC
1)**0.1
HITP=3.5*HIB/SQRT(XTT)
GT=4844.673*W
CALL ALPFA(XTT,GT,ALFA12)
CALL ALPFA(XTTPR,GT,ALFAPR)
ALFAP=0.5*(ALFA12+ALFAPR)
HINB=0.074*(TW1-TI)**2.86
HIB=ALFAM*HINB+HITP
65 U1B=(1./(1.25357/(TFETA(1)*HIB)+1./HOB+0.001315+0.002))/60.
L1S=HEIGHT(IPHASE,H,TBULK,TW1,TIN,W,IEFECT)
L1B=L1-L1S
DELT1S=TC-(TI+TIN)*0.5
Q1S=U1S*A *L1S*DELT1S/L1
DELT1B=TC-TI
Q1B=U1B*A *L1B*DELT1B/L1
Q=Q1S+Q1B
RETURN
2000 FORMAT(2X,'AFTER',I4,2X,'ITERATIONS IN HEAT THERE WAS NO ',
1'CONVERGENCE',/,2X,'ARGUMENTS WERE',/,5X,'TO=',F10.3,2X,
2'TI=',F10.3,2X,'TIN=',F10.3,'PHASE=',I4,2X,'EFFECT=',I4)
END

```

```

C *****
C                                     HEIGHT
C *****
C  PURPOSE
C    DETERMINES LIQUID HEIGHT IN EACH EFFECT
C  USAGE
C    FUNCTION HEIGHT(IPHASE,H,TX,T1BW,TIN,FLOW,IEFFECT)
C  PARAMETERS
C    IPHASE  - =1, SINGLE PHASE FLOW
C             - =2, TWO PHASE FLOW
C    H       - HOLD UP OF LIQUID (LBS)
C    TX      - MEAN LIQUID TEMPERATURE (DEG F)
C    T1BW    - INNER WALL TEMPERATURE. LIQUID SIDE. (DEG F)
C    TIN     - ENTERING LIQUID TEMPERATURE (DEG F)
C    FLOW    - FLOW RATE OF ENTERING LIQUID (LBS/MIN)
C    IEFFECT - =1, FIRST EFFECT
C             - =2, SECCND EFFECT
C  SUBPRCGRAMS REQUIRED
C    SUBRCUTINE SEARCH
C    SUBRCUTINE DET5 (FROM THE IBM SSP)
C
C  FUNCTION HEIGHT(IPHASE,H,TX,T1BW,TIN,FLOW,IEFFECT)
C  DIMENSION Z(20)
C  COMMON/CATA1/NCATA1,TEMP(22),THALP(22),THALPV(22),
1ALAMDA(22),PRESS(22),RHOV(22)
C  COMMON/CATA2/NCATA2,T(8),RHO(8),CP(8),VISC(8),THCOND(8),
1PRANTL(8),BETA
C  DATA NORDER/4/
C  AL=2.3125
C
C    TUBE DIAMETER IN BOTH EFFECTS = 0.0725 FT.
C
C  D=0.0725
C
C    NUMBER OF TUBES. FIRST EFFECT = 3.  SECOND EFFECT = 15.
C
C  NT=3
C  IF(IEFFECT.EQ.2)NT=15
C  TBULK=TX
C  CALL SEARCH(T,RHO,TBULK,NCATA2,NORDER,RHOF)
C  IF(IPHASE.EQ.2)GO TO 20
C  AT=3.14159*D*D/4.
C
C    DEAD HCLDUPS (DO NOT CONTRIBUTE TO HEIGHT).
C    FIRST EFFECT = 1.5315 LBS.  SECOND EFFECT = 16.336 LBS.
C
C  HCLD = F-1.5315
C  IF(IEFFECT.EQ.2)HOLD=F-16.336
C  HEIGHT=FCLD/(FLOAT(NT)*RHOF*AT)
C  IF(HEIGHT.LT.0.)HEIGHT=0.
C  IF(IEFFECT.EQ.2)GO TO 10

```

```

C
C      LENGTH OF TUBES. (ALSO MAXIMUM HEIGHT)
C      FIRST EFFECT =9.5 FT.  SECOND EFFECT = 1.917 FT.
C
IF(HEIGHT.GE.9.5)HEIGHT=9.5
RETURN

C
C      THIS SECTION OF THE SUBROUTINE DETERMINES THE LENGTH OF
C      THE SENSIBLE HEATING ZONE IN TWO-PHASE FLOW.  REFER
C      TO APPENDIX A FOR THE EQUATIONS AND REFERENCES
C
10 IF(HEIGHT.GE.1.917)HEIGHT=1.917
   RETURN
20 HL=FCIF(TBULK,FLOW)
   CALL SEARCH(T,CP,TBULK,NDATA2,NORDER,CPB)
   IF(FLW.EQ.0.)GO TO 30
   DTDL=(3.14159*D*FLOAT(NT)*HL*(T1BW-TBULK))/(60.*FLOW*CPB)
30 CPDL=-RHOE/144.
   CALL DET5(10.,PRESS,Z,NDATA1,IER)
   CALL SEARCH(TEMP,Z,TBULK,NDATA1,NORDER,DPDT)
   CTDG=1./CPCT
   IF(FLW.EQ.0.)GO TO 40
   HEIGHT=AL*CTDG/ (-(DTDL/CPDL)+DTDG)
   GO TO 50
40 DELT=2.*(TX-TIN)
   HEIGHT=1.+(DELT/(DPDL*DTDG))
50 IF(HEIGHT.GT.9.5)HEIGHT=9.5
   RETURN
END

```

```

C *****
C                                     FCINB
C *****
C      PURPOSE
C      CALCULATES THE INSIDE FILM COEFFICIENT FOR NATURAL
C      CONVECTION AND POOL BOILING USING THE RHOSENOW EQUATION
C      USAGE
C      FUNCTION FCINB(TW,TI)
C      PARAMETERS
C          TW      - INNER WALL TEMPERATURE, DEG F, INPUT
C          TI      - INNER BULK TEMPERATURE, DEG F, INPUT
C      SUBPROGRAMS REQUIRED
C          SUBROUTINE SEARCH
C
C      FUNCTION FCINB(TW,TI)
C      COMMON/DATA2/NDATA2,T(8),RHO(8),CP(8),VISC(8),THCOND(8),
1 PRANTL(8),BETA
C      COMMON/DATA1/NDATA1,TEMP(22),THALP(22),THALPV(22),
1 ALAMDA(22),PRESS(22),RHOV(22)
C      DATA NORDB/4/
C      DELTAT=TW-TI
C      IF(DELTAT.LE.0.)GO TO 10
C      TFILM=TW-0.75*(TW-TI)
C      CALL SEARCH(T,VISC,TI,NDATA2,NORDB,VISCF)
C      CALL SEARCH(TEMP,THALPV,TFILM,NDATA1,NORDB,HFG)
C      CALL SEARCH(T,RHO,TI,NDATA2,NORDB,RHOL)
C      CALL SEARCH(T,CP,TI,NDATA2,NORDB,CL)
C      CALL SEARCH(T,PRANTL,TI,NDATA2,NORDB,PRANTF)
C      CALL SEARCH(TEMP,ALAMDA,TFILM,NDATA1,NORDB,ALAMF)
C
C      CONVERT ALAMF TO CALS/G-MOLE
C
C      ALAMF=ALAMF*10.
C
C      CONVERT RHOL TO GMS/CC
C
C      RHCLB=RHOL*0.016
C
C      CALCULATE SURFACE TENSION FROM WALDEN'S EQUATION (PERRY)
C      IN DYNES/CM
C
C      ST=(ALAMF*RHCLB)/364.
C
C      CONVERT SURFACE TENSION TO LBS/FT
C
C      ST=ST*68.5218E-06
C      A=CL*DELTAT/HFG
C      B=ST/RHCL
C      FCINB=(VISCF*HFG*A*A*A)/(2.197E-06*DELTAT*SQRT(B)*PRANTF**5
C      .1)
C
C      RETURN

```

```
C
C      DELTAT IS NEGATIVE
C
10 FCIN2=0.
   RETURN
   END
```



```

C*****
C                                     FCIF
C*****
C  PURPOSE
C    CALCULATES THE INSIDE FILM COEFFICIENT DUE TO FORCED
C    CONVECTION USING THE DITTUS-BOELTER EQUATION
C  USAGE
C    FUNCTION FCIF(TBULK,W,TW1)
C  PARAMETERS
C    TBULK  - BULK TEMPERATURE OF LIQUID, DEG F, INPUT
C    W      - FLOW RATE OF LIQUID, LBS/HR, INPUT
C    TW1    - INNER WALL TEMPERATURE, DEG F, INPUT
C
C  FUNCTION FCIF(TBULK,W,TW1)
C    COMMON/DATA2/NDATA2,T(8),RHO(8),CP(8),VISC(8),THCOND(8),
C    1 PRANTL(8),BETA
C    DATA NORDER/4/
C    CALL SEARCH(T,RHO,TBULK,NDATA2,NORDER,RHOF)
C    CALL SEARCH(T,VISC,TBULK,NDATA2,NORDER,VISCF)
C    CALL SEARCH(T,CP,TBULK,NDATA2,NORDER,CPF)
C    CALL SEARCH(T,PRANTL,TBULK,NDATA2,NORDER,PRANTF)
C    CALL SEARCH(T,THCOND,TBULK,NDATA2,NORDER,THCF)
C    CALL SEARCH(T,VISC,TW1,NDATA2,NORDER,VISCW)
C    FLOW=60.*W
C    RE=4.8567*FLOW/VISCF
C    IF(RE.LE.2000.)FCIF=2.093414*THCF*RE**0.545*PRANTF**0.4*
C    1 (VISCF/VISCW)**0.14
C    IF(RE.LE.5000..AND.RE.GT.2000.)FCIF=0.88655*THCF*RE**0.66*
C    1 PRANTF**0.4*(VISCF/VISCW)**0.14
C    IF(RE.GT.5000.)FCIF=0.2745000*THCF*RE**0.8*PRANTF**0.4*
C    1 (VISCF/VISCW)**0.14
C    RETURN
C  END

```

```

C *****
C                                     FCO
C *****
C      PURPOSE
C          CALCULATES THE OUTSIDE (VAPOR SIDE) FILM COEFFICIENT USING
C          THE NUSSELT EQUATION
C      USAGE
C          FUNCTION FCO(TO,TW2,IEFECT)
C      PARAMETERS
C          TO      - MEAN VAPOR TEMPERATURE, DEG F, INPUT
C          TW2     - OUTER WALL TEMPERATURE, DEG F, INPUT
C          IEFECT  - PARAMETER INDICATING THE EFFECT
C      SUBPROGRAMS REQUIRED
C          SUBROUTINE SEARCH
C
C      FUNCTION FCO(TO,TW2,IEFECT)
C      COMMON/NDATA2,T(8),RHO(8),CP(8),VISC(8),THCOND(8),
1 PRANTL(8),BETA
C      COMMON/NDATA1,TEMP(22),THALP(22),THALPV(22),
1 ALAMDA(22),PRESS(22),RHOV(22)
C      DATA NORDER/4/
C      CALL SEARCH(TEMP,ALAMDA,TO,NDATA1,NORDER,ALAMF)
C      TFILM=TC-0.75*(TO-TW2)
C      CALL SEARCH(T,THCOND,TFILM,NDATA2,NORDER,THCF)
C      CALL SEARCH(T,RHO,TFILM,NDATA2,NORDER,RHOF)
C      CALL SEARCH(T,VISC,TFILM,NDATA2,NORDER,VISCF)
C      DELTAT=TC-TW2
C      AL=9.5
C      IF(IEFECT.EQ.2)AL=2.3125
C      G=4.17E08
C      IF(DELTAT.GT.0.)GO TO 10
C      FCO=0.1E+40
C      RETURN
10 FCO=0.943*(THCF**3*RHOF**2*G*ALAMF/(DELTAT*AL*VISCF))**0.25
C      RETURN
C      END

```

```

C *****
C                                     FCIN
C *****
C      PURPOSE
C          CALCULATES THE INSIDE FILM COEFFICIENT FOR NATURAL
C          CONVECTION USING THE NATURAL CONVECTION EQUATION
C      USAGE
C          FUNCTION FCIN(TBULK,TW,H,IEFFECT)
C      PARAMETERS
C          TBULK - BULK TEMPERATURE OF LIQUID, DEG F, INPUT
C          TW    - INNER WALL TEMPERATURE, DEG F, INPUT
C          H     - HOLD-UP, LBS, INPUT
C          IEFFECT - PARAMETER INDICATING THE EFFECT
C      SUBPROGRAMS REQUIRED
C          SUBROUTINE SEARCH
C          FUNCTION HEIGHT
C
C      FUNCTION FCIN(TBULK,TW,H,IEFFECT)
C      COMMON/ DATA1/ NDATA1,TEMP(22),THALP(22),THALPV(22),
1ALAMDA(22),PRESS(22),RHOV(22)
C      DATA NORDER/4/
C      TFILM=0.5*(TBULK+TW)
C      IF((TW-TBULK).LE.0.)GO TO 10
C      CALL SEARCH(T,THCOND,TFILM,NDATA2,NORDER,THCF)
C      CALL SEARCH(T,RHO,TFILM,NDATA2,NORDER,RHOF)
C      CALL SEARCH(T,VISC,TFILM,NDATA2,NORDER,VISCF)
C      CALL SEARCH(T,PRANTL,TFILM,NDATA2,NORDER,PRANTF)
C      XL=HEIGHT(1,H,TBULK,TW,0.,0.,IEFFECT)
C      IF(ABS(XL).LT..1E-70)GO TO 10
C      GR=(XL**3*RHOF*RHOF*4.17E8*BETA*(TW-TBULK))/(VISCF*VISCF)
C      Y=GR*PRANTF
C      IF((Y.GE.1.E4).AND.(Y.LT.3.5E7))FCIN=((0.55*THCF)/XL)
1*(Y**0.25)
C      IF(Y.GE.3.5E7)FCIN=((0.13*THCF)/XL)*(Y**0.333)
C      IF(Y.LT.1.E4)GO TO 2000
C      RETURN
2000 WRITE(6,2100)
C      FCIN=((0.55*THCF)/XL)*(Y**0.25)
C      RETURN
10 FCIN=C.
C      RETURN
2100 FORMAT(5X,'Y IS OUT OF RANGE')
C      END

```

```

C *****
C                                     MAP1
C *****
C      THIS SUBROUTINE MAPS THE ELEMENTS OF A VECTOR INTO
C      DESIRED POSITIONS IN ANOTHER VECTOR
C
C      USAGE
C      CALL MAP1(X,N,Y,IPOSIT,M,Z)
C
C      PARAMETERS
C      X      - INTO VECTOR INTO WHICH ELEMENTS ARE MAPPED
C      N      - DIMENSION OF X AND Z, INPUT
C      Y      - INPUT VECTOR TO REPLACE ELEMENTS IN X
C      IPOSIT - VECTOR CONTAINING INTEGERS INDICATING WHICH
C               ELEMENTS IN X ARE TO BE REPLACED BY THE
C               ELEMENTS OF Y
C      M      - DIMENSION OF Y AND IPOSIT, INPUT
C      Z      - OUTPUT VECTOR
C
C
C      SUBROUTINE MAP1 (X,N,Y,IPOSIT,M,Z)
C      DIMENSION X(1),Y(1),IPOSIT(1),Z(1)
C      DO 10 I=1,N
C      Z(I)=X(I)
10  CONTINUE
C      ICOUNT=1
20  J=IPOSIT(ICOUNT)
C      Z(J)=Y(ICOUNT)
C      ICOUNT=ICOUNT+1
C      IF(ICOUNT.LE.M)GO TO 20
C      RETURN
C      END

```

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BIOGRAPHICAL SKETCH

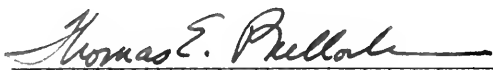
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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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Thomas E. Bullock
Associate Professor of Electrical Engineering

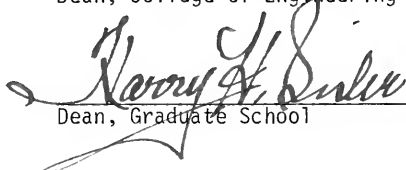
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March, 1974

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